ГЕОІНФОРМАТИКА

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THE STATISTICAL SIMULATION OF DATASET IN 3D AREA WITH "CUBIC" CORRELATION FUNCTION ON EXAMPLE RIVNE NPP GEOPHYSICAL MONITORING

(Представлено членом редакційної колегії д-ром геол. наук, проф. Б. П. Масловим)

Background. ***The model and algorithm were developed by using optimal in the mean square sense "cubic" correlation function. An example of supplementing the results of geophysical studies of karst-suffuses phenomena with simulated data in the task of monitoring the density of the chalk stratum on the territory of the Rivne NPP is presented.***

The complex geophysical research was conducted on Rivne NPP area. The monitoring observations radioisotope study of soil density and humidity near the perimeter of buildings is of the greatest interest among these. In this case a problem was occurred to supplement simulated data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells.

This problem was solved in this work by statistical simulation method that provides the ability to display values (the random field of a research object in 3D area) in any point of the monitoring area.

Methods. ***Based on the spectral decomposition of random fields in 3D space, a statistical model of the distribution of the average density of the chalk layer in the 3D observation area was built.***

Results. ***An algorithm for statistical simulation of random fields with a "cubic" correlation function is formulated. On the basis of the developed software, additional simulated realizations of the random component of the research subject on the grid of observations of the necessary detail and regularity were obtained. A statistical analysis of the results of the numerical simulation of the distribution of the average density of the chalk layer was carried out and their adequacy was tested.***

Conclusions. ***The method of statistical modeling of random fields with "cubic" correlation functions allows you to supplement data with a given accuracy.***

Keywords: ***Statistical simulation, "cubic" correlation function, spectral decomposition, conditional maps.***

Background

Due to the increasing number of dangerous natural and technogenic disasters in the world, the development of geological environment monitoring system is actual using modern mathematical tools and information technology. The regular local monitoring of potentially dangerous objects of human activity is an important part of the overall environment monitoring system. When monitoring such objects, many actual problems were raised, for example, such as the lack of some data in the database, or insufficient quantity or necessity to supplement the database on the grid with required detail without conducting additional research.

Theoretical aspects of capacity use of the statistical simulation methods based on spectral decomposition to solving different apply problems considered in the works (Yadrenko, 1983; Guyon, 1993; Chiles, & Delfiner, 2012; Vyzhva, 2003, 2011, 2021). Practical testing on real density chalky strata data on the Rivne NPP territory was carried out for the 3D area – in the following works by using Bessel correlation function (Vyzhva, Demidov, & Vyzhva, 2013), Cauchy correlation function (Vyzhva, Demidov, & Vyzhva, 2014b) and spherical correlation function (Vyzhva, Demidov, & Vyzhva, 2020b).

In this paper, the statistical simulation method for random field in 3D area we propose to use with the model and procedure involving enough adequate in the mean square sense data "cubic" correlation function**.** It is known (Chiles et al., 2005), that the "cubic" covariance model is used for differentiable variables as well as potential fields in geological modelling. The potential-field method was designed by (Chiles et al., 2005) to build 3D geological models from data available in geology and mineral exploration, namely the geological map, structural data, borehole data and interpretations of the geologist, because most 3D geological known modelling tools were designed for the needs of the oil industry and are not suited to the variety of situations arisen in other application domains of geology. This problem was considered also in paper (Vyzhva, Demidov, & Vyzhva, 2020a).

Note, that methods of 3D random fields statistical simulation used in geosciences problems was developed by the scientists: Mantoglov, & Wilson, 1981; Wackernagel, 2003; Emery, & Lantuejoul, 2006; Webster, & Oliver, 2007; Chiles, & Delfiner, 2012; Tolosana-Delgado, & Mueller, 2021 and other.

***The essential problems of karst-suffusion phenomena monitoring at Rivne NPP territory*.** The problems of the random field simulation in 3D area arise in solving the actual environmental geophysical monitoring problems. The complex geophysical research was conducted on Rivne NPP territory during many years. The radioisotope study of soil density and humidity near the perimeter of constructed buildings on object is of the greatest interest among these monitoring observations. The soil density was determined by gamma-gamma well logging, soil humidity was determined by neutron-neutron logging, when conducting this research.

In this case (Vyzhva, Demidov, & Vyzhva, 2013) a problem occurred to supplement adequately simulated adequate data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells. Schematic representation of the measurement results chalky strata density at the Rivne NPP object that was investigated, and the well locations are shown on Fig. 1. These data are obviously not enough in detail to represent the overall picture of the chalk strata on object, where due to the aggressive water action the karst-suffusion processes were significantly intensified.

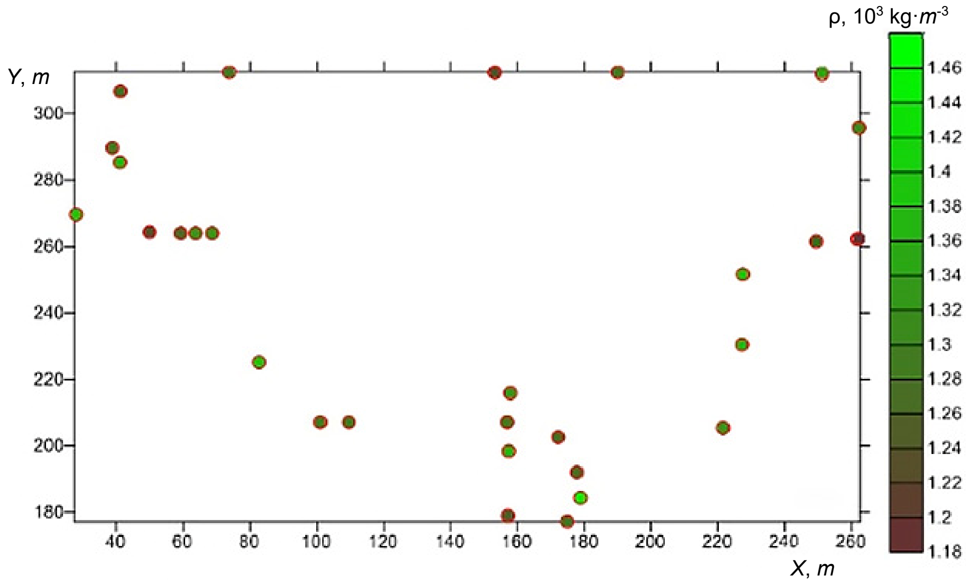

Fig. 1. Observation points and chalk strata averaged density at industrial area of Rivne NPP

This noted problem was solved in following works: (Vyzhva, Demidov, & Vyzhva, 2013, 2014a, 2014b, 2019, 2020b) by statistical simulation method that provides the ability to display values (random field in three-dimensional space) in any point of the monitoring area. The chalk strata averaged density at the industrial area was simulated using the built 3D model and the involvement of the Bessel type correlation function (Vyzhva, Demidov, & Vyzhva, 2013), Cauchy correlation function (Vyzhva, Demidov, & Vyzhva, 2014b) and spherical correlation function (Vyzhva, Demidov, & Vyzhva, 2020b).

This paper continues further development of 3D statistical simulation methods, involving "cubic" correlation function that is well-known in geostatistic works (Chiles et al., 2005; Chiles, Delfiner, 2012; Vyzhva, Demidov, & Vyzhva, 2020a). This operation was done for data array of density chalk strata in the years 1984–2002 for 29 wells at Rivne NPP industrial area and depths are 28 m, 29 m and 30 m below the surface. The difference between the card input density values and the trend is in most cases a homogeneous isotropic random field (Vyzhva, Demidov, & Vyzhva, 2014b, 2019). The stationary random component in the three-dimensional area is proposed to modeling on the basis of spectral decomposition (Vyzhva, 2003, 2011) with "cubic" correlation function in this paper.

The statistical method of solving the problems at Rivne NPP territory. The statistical simulation of density chalky strata data at the Rivne NPP object was performed at three levels (28, 29, 30 meters from the surface) in this paper. While constructing graphs of density chalky strata data for each specified account, we noticed that it is expedient to distinguish deterministic and random components. Deterministic component can be selected as function by the method of approaching the minimum curve (separation of the so-called trend). The difference between the map of input density values and the trend is a realization of homogeneous isotropic random field in the most cases, as we mentioned above and which is very important.

Input data on the each of three level from the surface is a realization of random field in 3D space $\eta(x, y, z_i),\ i = 1,2,3$; $z_1 = 28m,\ z_2 = 29m,\ z_3 = 30m.$ $\eta(x, y, z_i) = \eta_i(r, \theta, \varphi)$, (r, θ, φ) – spherical coordinates, i – level numbers. The trend $f_i(r, \theta, \varphi)$ and the random component $\xi_i(r, \theta, \varphi)$ (frequently homogeneous isotropic random field in 3D space, so-called "noise") were selected for each level:

$$\eta_i(r, \theta, \varphi) = f_i(r, \theta, \varphi) + \xi_i(r, \theta, \varphi), \qquad i = 1, 2, 3.$$

The imposing array of random field realizations $\xi_i(r, \theta, \varphi)$ $(i = 1,2,3)$ was got by statistical simulation in add points from three-dimensional observations area for the approximation of real data, we superimpose on the trend $f_i(r, \theta, \varphi),\ i = 1,2,3$. As a result of final modelling stage, we received more detailed addition for the chalk layer density data in the selected three-dimensional area at Rivne NPP.

We use the method of statistical simulation of homogenous isotropic random fields in 3D area for the solving arising problem, which is based on their spectral decomposition (Vyzhva, 2003). This technique allows to find the sufficiently perfect image of input data random fields in the whole observation area at the Rivne NPP by means of simulated realizations of random component of this data.

At first it is necessary to make the statistical analysis of data before building the model and procedure of chalky strata density simulation at observation 3D area. If the three-dimensional input data has distribution density with approximately Gaussian type, then procedure can be used, which is developed in (Vyzhva, Demidov, & Vyzhva, 2013; Vyzhva, 2011), to generate on the computer realizations of the simulated data by means of sequences standard normal random variables.

The distribution of ***chalky*** strata density data at the ***Rivne NPP*** is determined. The preliminary statistical analysis of 3D input data shows that the distribution histogram of chalky strata density at the Rivne industrial area (29 boreholes) approximately has Gaussian distribution (Fig. 2).

The use of authors' techniques of statistical simulation involves preliminary statistical processing of data to determine their statistical characteristics, in particular, the correlation function model. If the hypothesis of Gaussian distribution of the investigated data field is confirmed, then the mathematical expectation and the correlation function completely define this random field in three-dimensional area and give us the opportunity to build the adequate statistical model for data field, which is based on the spectral decomposition. The principles of constructing the models and statistical simulation procedures for the "cubic" correlation function were considered in work (Vyzhva, Demidov, & Vyzhva, 2020a), and described below.

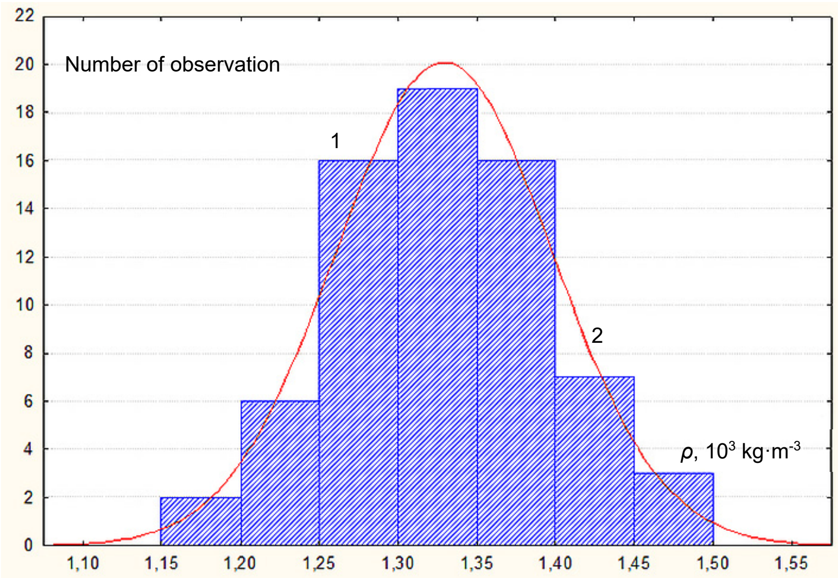

Fig. 2. Histogram of the chalky strata density (averaged data for all years of observation):
1 – the number of observations in a separate range of density; 2 – theoretical Gaussian curve

Then for the data correlation function $B(\rho)$ (ρ) –t he distance between vectors $x, y \in R^3$ $(x = (r_1, \theta_1, \varphi_1)$, $y = (r_2, \theta_2, \varphi_2))$ statistical models were chosen for the distribution of the density of chalk strata in the three-dimensional observation area. This function is determined by comparing the root-mean-square approximation of empirical and theoretical variograms of the density data of chalk layers.

As a result, the input data was most adequately described using of 3 types of correlation functions: the Bessel correlation function (1) at the value of parameter $c = 5$, the Cauchy correlation function (2) at the value of parameter $a = 1$, the spherical correlation function (3) at the value of parameter $a \approx 1,25 * 10^{-2}$ and the "cubic" correlation function (4) at the value of parameter $a \approx 1,25 * 10^{-2}$.

$$B(\rho) = \sqrt{\frac{\pi}{2\,cr}}\, J_{\frac{1}{2}}(c\rho), \quad c = 5, \tag{1}$$

where $J_k(x)$ is the Bessel function of the first kind of order $k = 1/2$,

$$B(\rho) = \frac{a^4}{(a^2+\rho^2)^2}, \quad a = 1, \tag{2}$$

$$B(\rho) = \begin{cases} 1 - \frac{3}{2}\,\frac{\rho}{a} + \frac{1}{2}\left(\frac{\rho}{a}\right)^3, & \rho \le a; \\ 0, & \rho > a. \end{cases} \tag{3}$$

$$B(\rho) = \begin{cases} 1 - 7\left(\frac{\rho}{a}\right)^2 + \frac{35}{4}\left(\frac{\rho}{a}\right)^3 + \frac{7}{2}\left(\frac{\rho}{a}\right)^5 + \frac{3}{4}\left(\frac{\rho}{a}\right)^7, & \rho \le a; \\ 0, & \rho > a. \end{cases} \tag{4}$$

It is known, that homogenous isotropic random field has a variogram $\gamma(\rho)$. In that case the variogram (Chiles et al., 2005) is linked to the correlation function $B(\rho)$ by the relation

$$\gamma(\rho) = B(0) - B(\rho). \tag{5}$$

Thus, the variogram of a homogenous isotropic random field is bounded by 2 $B(0)$. Equation (5) shows that if the correlation function is known, the variogram is also known. Conversely, if the variogram of a homogenous isotropic random field is bounded by a finite value, $\gamma(\rho)$ is of the form (5). If the variogram has a sill, the value of $B(0)$ must be chosen equal to or greater than the sill. It is then equivalent to know $\gamma(\rho)$ or $B(\rho)$.

Variograms of input chalky strata density 3D data at the Rivne NPP was built by using the *R* software and geo*R* package They corresponding to the: Bessel (1) correlation function (the mean square approximation is 0,0008599), Cauchy (2) correlation function (the mean square approximation is 0,002816), spherical (3) correlation function (the mean square approximation is 0,000480) and "cubic" correlation function (4) (the mean square approximation is 0,001360). Variograms plots were presented at Fig. 3a, according to Bessel type of correlation function, at Fig. 3b, according to Cauchy type of correlation function, at Fig. 3c, according to spherical type of correlation function for the random component of investigation three-dimensional data.

The built empirical variogram (Fig**.** 4) of input chalky strata density 3D data at the Rivne NPP has the best approximation by theoretical variogram which is connected to the "cubic" correlation function (4) with parameter $a \approx 1,25 * 10^{-2}$.

We have, that the constructed variogram of realizations in the studied territory (Fig. 5) has sufficiently adequate approximation by the theoretical variogram, which is associated with a "cubic" correlation function (a mean square deviation is 1,36 * 10^{-3}).

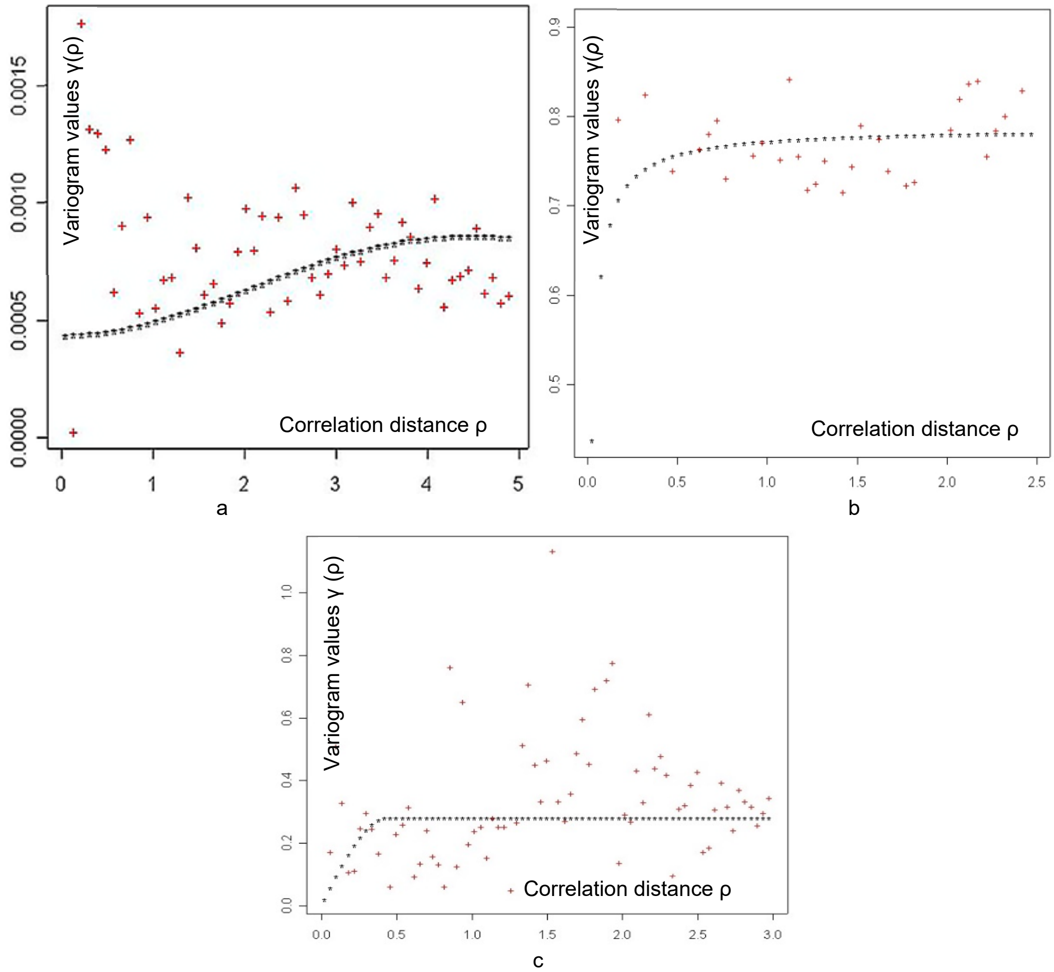

Fig. 3. Empirical (crosses) and theoretical (curve) variograms for input data of the chalky strata, that corresponding to the:
a – the Bessel (1) correlation function; b – the Cauchy (2) correlation function; c – the Spherical (3) correlation function

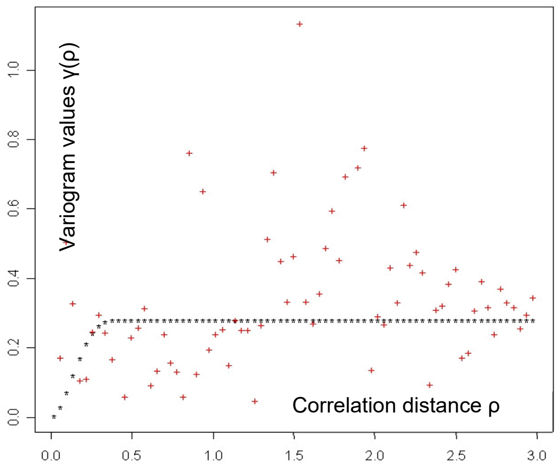

Fig. 4. Empirical (crosses) and theoretical (curve) variograms of input data arrays of chalk layer density, corresponding to "cubic" correlation function $(a \approx 1,25 * 10^{-2})$

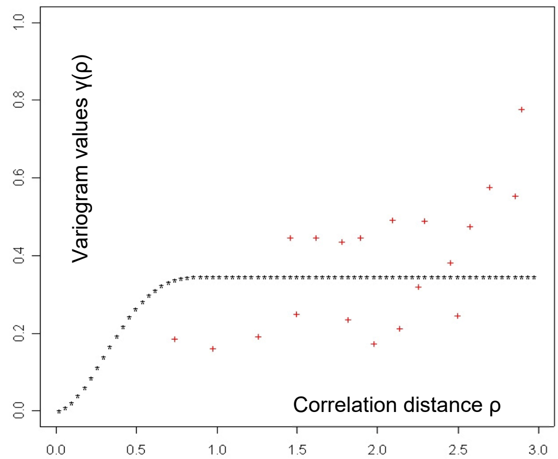

Fig. 5. Empirical (crosses) and theoretical (curve) variograms of simulated data arrays of chalk layer density, corresponding to "cubic" correlation $(a \approx 1,25 * 10^{-2})$

Methods

The spectral representation of homogeneous isotropic random fields in the 3D area, approximation theorem, model. Now we present some theorems from the spectral theory of random fields. We consider a real-valued homogeneous isotropic random field $\xi(r, \theta, \varphi)$ in the three-dimensional area ($\xi(r, \theta, \varphi)$ – spherical coordinates). It is known (Yadrenko, 1983; Vyzhva, 2003; Vyzhva, 2011, p. 208) that square-mean continuous real-valued isotropic random field $\xi(r, \theta, \varphi)$, that is in 3D Euclidean space R^3, admit the spectral decomposition by spherical harmonics.

The correlation function of the homogeneous isotropic random field $\xi(r,\theta,\varphi)$ in three-dimensional area $B(\rho)$ depends on distance ρ between the vectors $x, y \in R^3$ $(x = (r_1, \theta_1, \varphi_1),\ y = (r_2, \theta_2, \varphi_2)$:

$$\rho = r\sqrt{2(1-\cos\psi)} = r\sin(\psi/2),$$

where $\cos\psi$ – angular distance between vectors $x, y \in R^3$:

$$\cos\psi = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2).$$

However, the spectral decomposition of this random field is used to solve the problems of statistical modeling of realizations of a random field in three-dimensional space, where real-valued random variables occur. Let's add this decomposition.

Theorem 1. Let a mean square continuous realvalued homogeneous isotropic random field $\xi(r,\theta,\varphi)$ is in 3D space with zero mean. Then this random field admits (Vyzhva, 2011, p. 210) the following spectral decomposition:

$$\xi(r,\theta,\phi) = \sum_{m=0}^{\infty}\sum_{l=0}^{m}\tilde{c}_{m,l}\,P_m^l(\cos\theta)\,[\varsigma_{m,1}^l(r)\cos l\phi + \varsigma_{m,2}^l(r)\sin l\phi], \qquad (6)$$

where P_m^l is associated Legendre functions of degree m, $\tilde{c}_{m,l}$ – constants sequences are calculated by the formula:

$$\tilde{c}_{m,l} = \frac{1}{2}\sqrt{\frac{\nu_l}{\pi}\frac{(m-l)!}{(m+l)!}(2m+1)}\,, \quad \nu_l = \begin{cases}1, & l \neq 0,\\ 2, & l = 0;\end{cases} \qquad (7)$$

random processes sequences $\{\varsigma_{m,k}^l(r)\}$, $k = 1,2$: $\varsigma_{m,k}^l(r) = \int_0^\infty \frac{J_{m+\frac{1}{2}}(\lambda r)}{(\lambda r)^{\frac{1}{2}}} Z_{m,k}^l(d\lambda)$, satisfying the following conditions:

1) $M\ \varsigma_{m,k}^l(r) = 0$;

2) $M\ \varsigma_{m,k}^l(r)\ \varsigma_{m',k'}^{l'}(r) = \delta_l^{l'}\ \delta_m^{m'}\ \delta_k^{k'}\ b_m(r)$, (8)

where $\delta_m^{m'}$ – Kronecker symbol, $b_m(r)$ – the spectral coefficients and $\{ Z_m^l(.) \}$ is a sequence of orthogonal random measures on Borel subsets from the interval $[0,+\infty)$, i.e.

$$E\ Z_m^l(S_1)\ Z_{m'}^{l''}(S_2) = \delta_l^{l'}\ \delta_m^{m'}\ \Phi(S_1 \cap S_2)\ ,$$

for any Borel subsets S_1 and S_2,

where $\Phi(\lambda)$ is the bounded nondecreasing function so-called spectral function of random field $\xi(r,\theta,\varphi)$.

The spectral density of homogeneous isotropic random field $\xi(r,\theta,\varphi)$ is defind as $f(\lambda) = d\Phi(\lambda)/d\lambda$ and it is obtained by correlation function of this random field as integral:

$$f(\lambda) = \frac{2}{\pi}\int_0^\infty \rho\lambda\sin(\lambda\rho)\,B(\rho)\,d\rho. \qquad (9)$$

The spectral coefficients $b_m(r)$ of random field $\xi(r,\theta,\varphi)$ are defined by the spectral density $f(\lambda)$ of this random field in three-dimensional space in the way:

$$b_m(r) = \int_0^\infty \frac{J^2_{m+\frac{1}{2}}(\lambda r)}{\lambda r} f(\lambda)d\lambda. \qquad (10)$$

Further the statistical simulation of homogeneous isotropic random fields in the 3D space on the basis the spectral decomposition (6) coefficients (10) are considered.

Approximation model for the homogeneous isotropic random field $\boldsymbol{\xi(r,\theta,\varphi)}$ is built by using the partial sums of series (6) and is presented by the formula:

$$\xi_N(r,\theta,\varphi) = \sum_{m=0}^{N}\sum_{l=0}^{m} c_{ml}\ P_m^l(\cos\theta)\left[\zeta_{m,1}^l(r)\cos l\phi + \zeta_{m,2}^l(r)\sin l\phi\right],\ N \in \mathrm{N}. \qquad (11)$$

We need the mean square approximation of random field $\xi(r,\theta,\varphi)$ by model (11) in the convenient form for the constructing statistical simulation of homogeneous isotropic random field realizations in the 3D space algorithm.

Further we used mean square estimate from the paper (Vyzhva, Demidov, & Vyzhva, 2018), what we have in following theorem.

Theorem 2. Let a mean square continuous realvalued isotropic random field $\xi(r,\theta,\varphi)$ on the sphere $S_3(r)$ in 3D space with zero mean (r – radius of sphere). If $\mu_3 < +\infty$, then the mean square approximation of this random field by model (9) is such that

$$M\ [\xi(r,\theta,\varphi) - \xi_N(r,\theta,\varphi)]^2 \le \frac{5\pi r^3}{2N^2}\mu_3, \qquad (12)$$

where $\mu_3 = \int_0^\infty \lambda^3\Phi\ (d\lambda)$. (13)

If we proposed, that r (radius of sphere) is not fixed, then the random field $\xi(r,\theta,\varphi)$ is in 3D Euclidean space R^3.

Further, we described the algorithm for the statistical simulation of realizations of Gaussian homogeneous isotropic random fields $\xi(r,\theta,\varphi)$ in 3D Euclidean space R^3, which was constructed on the basis of model (11) and estimate (12)

We constructed on this paper the algorithm for the statistical simulation of Gaussian homogeneous isotropic random fields on three-dimensional space space with "cubic" correlation function (4).

The spectral density is obtained for "cubic" correlation function (4) by means the formula (9) as:

$$f(\lambda) = \frac{2}{\pi}\int_0^a \rho\lambda\sin(\lambda\rho)\left(1 - 7\left(\frac{\rho}{a}\right)^2 + \frac{35}{4}\left(\frac{\rho}{a}\right)^3 + \frac{7}{2}\left(\frac{\rho}{a}\right)^5 + \frac{3}{4}\left(\frac{\rho}{a}\right)^7\right)d\rho. \qquad (14)$$

The spectral coefficients, which correspond to the "cubic" correlation function (4) of homogeneous isotropic random field $\xi(r,\theta,\varphi)$, are calculated by the formula (9) and we have:

$$b_m(r) = \frac{2}{\pi}\int_0^\infty \frac{J^2_{m+\frac{1}{2}}(\lambda r)}{\lambda^2 r}\left[\int_0^a \rho\lambda\sin(\lambda\rho)\left(1 - 7\left(\frac{\rho}{a}\right)^2 + \frac{35}{4}\left(\frac{\rho}{a}\right)^3 + \frac{7}{2}\left(\frac{\rho}{a}\right)^5 + \frac{3}{4}\left(\frac{\rho}{a}\right)^7\right)d\rho\right]\ d\lambda. \qquad (15)$$

These spectral coefficients $b_m(r), m = 0,1,2,\dots N$ are calculated by Mathematica software for density chalky strata data.

Results

The statistical simulation procedure of random field in 3D area with the "cubic" correlation function. In this paper we generated the realizations of homogeneous isotropic random field in 3D area with the "cubic" correlation function (4) at the values of parameter $a \approx 1,25 * 10^{-2}$. The statistical simulation of density chalky strata data at the Rivne NPP object was performed by the technique of spectral decomposition and finding of spectral coefficients.

The procedure of numerical simulation of the realizations of the 3D data field random component, by means of the above mentioned model (11), was conducted, which is described in (Vyzhva, Demidov, & Vyzhva, 2018).

The value of number N for the constructed model is determined by the inequality, which is the estimate of the mean square approximation of random field $\xi(r,\theta,\varphi)$ by partial sums $\xi(r,\theta,\varphi)$. This number N corresponds to the prescribed small number ε (approximation accuracy). The mentioned inequality was obtained in theorem 2. Consequently, the estimate of the mean square approximation of the random field $\xi(r,\theta,\varphi)$ with "cubic" type correlation function (4) by the partial sums $\xi(r,\theta,\varphi)$ has the following representation:

$$M\,[\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)]^2 \le \frac{5\pi r^3}{2N^2}\mu_3,$$

where

$$\mu_3=\frac{2}{\pi}\int_0^K \lambda^3\,[\int_0^a \rho\lambda\sin(\lambda\rho)\,(1-7\left(\frac{\rho}{a}\right)^2+\frac{35}{4}\left(\frac{\rho}{a}\right)^3+\frac{7}{2}\left(\frac{\rho}{a}\right)^5+\frac{3}{4}\left(\frac{\rho}{a}\right)^7)\,d\rho]\,d\lambda, K=K(a)-\text{const.} \quad (16)$$

We define dependence number N on r and ε in the case of-"cubic" correlation function (4) as a following:

$$N(r,\varepsilon)\ge\sqrt{\frac{5\pi r^3}{2\varepsilon}\mu_3}. \quad (17)$$

The statistical simulation procedure of Gaussian homogeneous isotropic random field $\xi(r,\theta,\varphi)$ in 3D area with "cubic" correlation function (4) was built by means of the model (9) and the estimate (15). This random field is determined by its statistical characteristics: the mathematical expectation and the "cubic" correlation function $B(\rho)$ (4) at the value of parameter $a \approx 1,25*10^{-2}$.

Algorithm:

1. Natural number N (border of summation) is chosen according to necessary accuracy $\varepsilon>0$ of approximation the model (11) mentioned below:

$$\frac{5\pi r^3}{2N^2}\mu_3 \le \varepsilon, \quad (18)$$

where

$$\mu_3=\frac{2}{\pi}\int_0^K \lambda^3\,[\int_0^a \rho\lambda\sin(\lambda\rho)\left(1-7\left(\frac{\rho}{a}\right)^2+\frac{35}{4}\left(\frac{\rho}{a}\right)^3+\frac{7}{2}\left(\frac{\rho}{a}\right)^5+\frac{3}{4}\left(\frac{\rho}{a}\right)^7\right)d\rho]\,d\lambda, K=K(a)-\text{const.}$$

Calculate the spectral coefficients $b_m(r), m=0,1,2,\dots N$ for the "cubic" correlation function (4) as integral (15).

2. Simulate the sequences of independent Gaussian normal random variables:

$\left\{\varsigma_{m,k}^l(r)\right\}$, $k=1,2;\ m=0,1,2,\dots N;\ l=1,\dots,m;$ that satisfying the following conditions (8) with spectral coefficients (15).

3. Calculate the realization of the stochastic random field $\xi(r,\theta,\varphi)$ by formula (11) in given point (r_i,θ_j,φ_p), $i=1,2,\dots,I;\ j=1,2,\dots,G;\ p=1,2,\dots,P$ in the 3D observations area by means of substituting in it values from the previous items 1, 2 and 3, numbers N and sequences of Gaussian random variables.

4. Check whether the realization of the random field $\xi(r,\theta,\varphi)$ generated in step 4 fits the data by testing the corresponding statistical characteristics (distribution and correlation function).

The statistical simulation of realizations of the Gaussian isotropic random fields $\xi(r,\theta,\varphi)$ with "cubic" correlation function can be done by means of this algorithm.

Note that the procedure can be applied to random fields with different type of distribution. Then the sequences of random variables $\left\{\zeta_{k,i}(r), i=1,2; k=0,1,2,\dots,N(r,\varepsilon)\right\}$ must be distributed according to the appropriate distribution type.

The original Spectr software, based on the results of the statistical data processing and the mentioned algorithm for the simulation values of such data realization in the 3D area, was developed in Python, where selected "cubic" type correlation function (4) was used. We calculate realizations of the random field $\xi(r,\theta,\varphi)$ in 100 points for each of 3 observations levels (r_i,θ_j,φ_p), $i=1,2,\dots,I;\ j=1,2,\dots,G;\ p=1,2,\dots,P$ in three-dimensional area by means of this software. Based on these realizations, a statistical estimate of the correlation function was obtained. This estimate compares with a given "cubic" type correlation function (4) at the value of parameter $a \approx 1,25*10^{-2}$ and provides a statistical analysis of the adequacy of realizations. The constructed variogram of these realizations on the studied area (Fig. 6) has an adequate approximation by theoretical variogram, which is associated with a "cubic" correlation function. The results show that the chosen data model on the density of chalk layers at the Rivne NPP site is quite adequate. The Spectr software developed for generating such realizations of the random field works with sufficient accuracy.

The results, obtained using additional modeling procedures, are shown in Fig. 6 a presents an example of constructed chalky strata density map according to observations data boreholes (averaged data over the years to 29 boreholes at a depth of 28 m) by Surfer software. Using available data, the accuracy of this design cannot provide a reliable characterization of the state of the chalk layers, since the number of measurement results is insufficient.

In Fig. 6 b presents contours of equal chalky strata density values based on simulation data, including values of anchor boreholes by calculating spectral coefficients of the "cubic" type. In addition, the output data (100 simulated values in the intervals between observation points of this level) can have more reliable approximation, that allows for more informed decisions about the state of chalky strata and identifies places for testing and additional research.

The following results, which were obtained using the simulating procedure for boreholes of observations data (averaged data for years up to 29 boreholes at a depth of 29 m, are displayed in Fig. 7. In Fig. 7a presents an example of a constructed map of the density of chalky strata according to observations for this data by Surfer software. In Fig. 7b presents contours of equal density values of chalky strata, which are based on simulation data, including values of the anchor boreholes. In addition, the output data (100 simulated values in intervals between observation points of this level) can have a more reliable approximation, which allows making more informed decisions about the state of chalky strata.

Finally, the results, that were obtained using the simulation procedure for boreholes of the observational data (averaged data over the years for 29 boreholes at a depth of 30 m, are shown in Fig. 8. Fig. 8 a shows an example of constructed map of the density of chalky strata according to observations of these data by the Surfer software. In Fig. 8b presents contours of equal density values of chalky strata, which are based on simulation data, including values of anchor boreholes. In addition, the output data (100 simulated values in the intervals between the observation points of this level) can have a more reliable approximation, which allows making more informed decisions about the state of chalky strata.

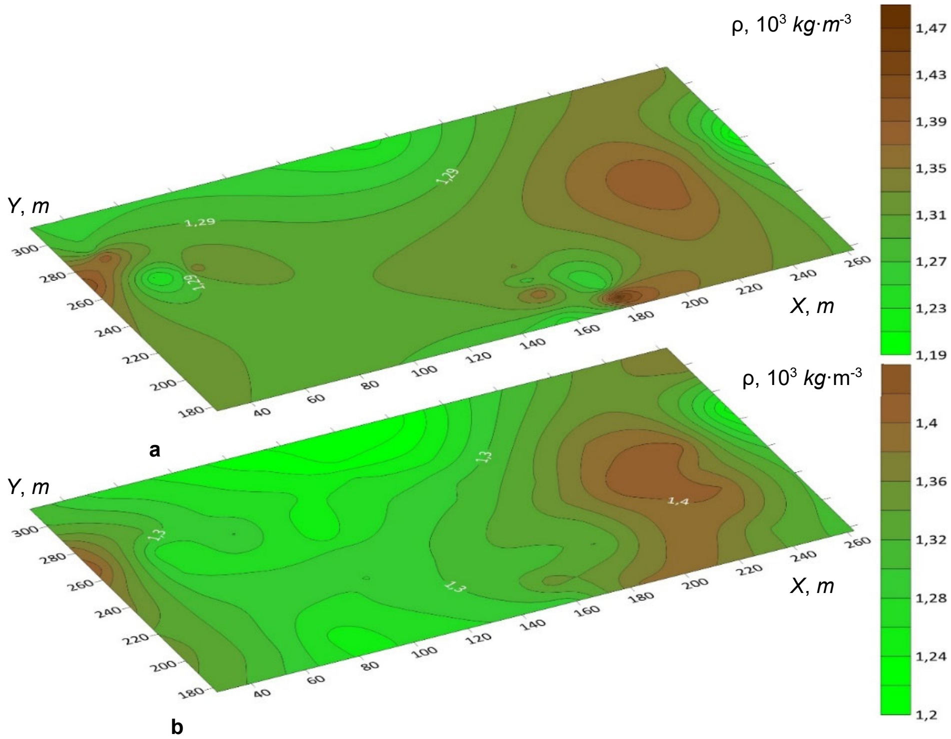

Fig. 6. The distribution of chalky strata density is on the industrial area of Rivne nuclear power plant at a depth of 28 m from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years, for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the "cubic" type

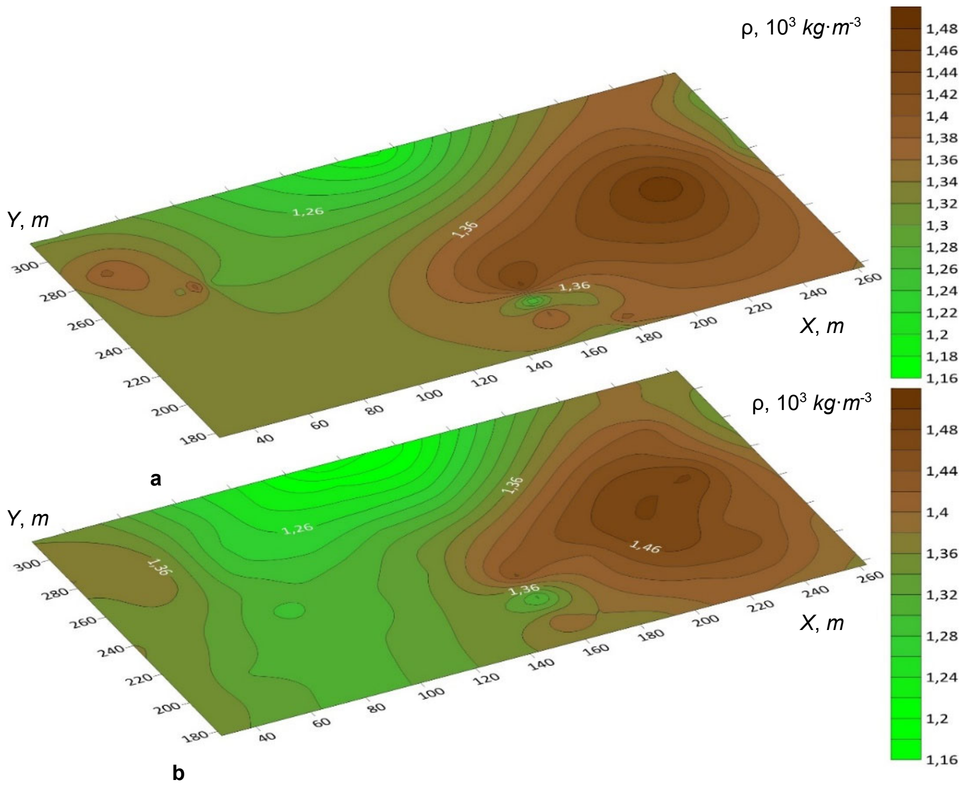

Fig. 7. The distribution of chalky strata density is on the Rivne NPP object at a depth of 29 m from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years, for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the "cubic" type

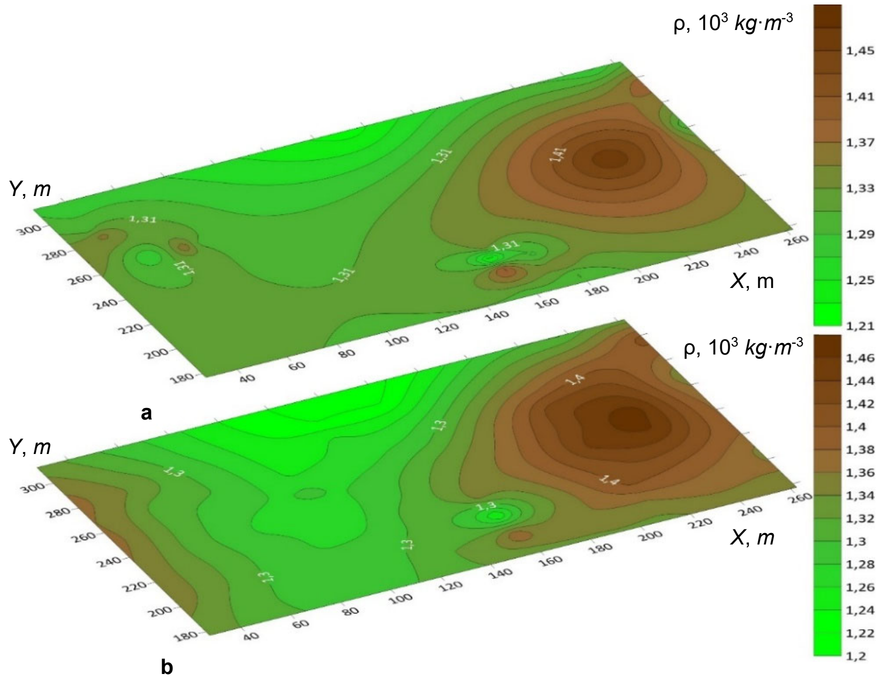

Fig. 8. The distribution of chalky strata density is on the industrial area of Rivne nuclear power plant at a depth of 30 m from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years, for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the "cubic" type

The statistical evaluation of the correlation function was obtained for the simulated realizations of the density distribution data of chalky strata in the 3D zone at the Rivne NPP facility. This estimate compares with a given theoretical "cubic" correlation function (4) at the value of parameter $a \approx 1,25 * 10^{-2}$ and ensures the adequacy of realizations on their statistical analysis. The constructed variogram of these realizations in the studied area (Fig. 5) has an adequate approximation by the theoretical variogram, which is associated with a "cubic" correlation function. The results show that the chosen model of the correlation function of the random component of the data is sufficiently adequate and the developed Spectr software for generating realizations works with sufficient accuracy.

Discussion and conclusions

The theory, methodology and procedure of statistical simulation of random fields in a three-dimensional area using the optimal in the mean square sense "cubic" correlation function allows significantly increasing the effectiveness of monitoring observations on the territory of potentially dangerous objects. This makes it possible to simulate values in the area between regime observation grids and abroad, more adequately describe 3D density data of chalky strata in the industrial area of Rivne nuclear power plant. It should be noted that the variogram of input data has the best approximation of the theoretical variogram, which is associated with a "cubic" correlation function with a mean square deviation of 0,00136, compared to the mean square approximation for the Cauchy correlation function of 0,002816.

The method of statistical simulation of random fields with the "cubic" correlation functions allows supplementing the data with a given accuracy. It can also be used to detect abnormal areas when studying geophysical parameters in three-dimensional space.

There are many other areas of statistical simulation of random fields in a three-dimensional area methods application in geosciences. Among them primary are soil science and environmental magnetism (Menshov et al., 2015).

Authors' contribution: Zoya Vyzhva – problem formulation, development of analytical expressions for the model, selection of scientific novelty, formal analysis, methodology, review and editing; Vsevolod Demidov – review of publications, data processing, conclusions, refinement and editing; Andriy Vyzhva – statistical modeling algorithm development, editing.

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СТАТИСТИЧНЕ МОДЕЛЮВАННЯ ДАНИХ У 3-D ОБЛАСТІ З "КУБІЧНОЮ" КОРЕЛЯЦІЙНОЮ ФУНКЦІЄЮ НА ПРИКЛАДІ ГЕОФІЗИЧНОГО МОНІТОРИНГУ РІВНЕНСЬКОЇ АЕС

В с т у п . ***Розроблено модель й алгоритм статистичного моделювання даних у 3D області з використанням оптимальної в середньому квадратичному наближенні "кубічної" кореляційної функції. Наведено приклад доповнення змодельованими даними результатів геофізичних досліджень карстово-суфозійних явищ у задачі моніторингу густини крейдяної товщі на території Рівненської АЕС.***

На території розміщення Рівненської АЕС проведено комплекс геофізичних досліджень. Серед таких моніторингових спостережень найбільший інтерес викликають радіоізотопні дослідження густини та вологості ґрунтів за периметром збудованих споруд. При цьому виникла проблема доповнення моделюванням даних, які отримано під час контролю зміни густини крейдяної товщі на території досліджуваного проммайданчика з використанням радіоізотопних методів по сітці, що охоплювала 29 свердловин.

М е т о д и . ***Зазначену проблему в роботі було розв'язано методом статистичного моделювання, що надає можливість відображати явище (випадкове поле об'єкта дослідження у тривимірній області) у будь-якій точці області спостереження. На основі спектрального розкладу випадкових полів у 3D-просторі побудовано статистичну модель розподілу усередненої густини крейдяної товщі у 3D області спостереження.***

Р е з у л ь т а т и . ***Сформульовано алгоритм статистичного моделювання випадкових полів з "кубічною" кореляційною функцією. Отримано на базі розробленого програмного забезпечення додатково змодельовані реалізації випадкової складової предмета дослідження на сітці спостережень необхідної детальності та регулярності. Проведено статистичний аналіз результатів чисельного моделювання розподілу усередненої густини крейдяної товщі та їхню перевірку на адекватність.***

В и с н о в к и . ***Метод статистичного моделювання випадкових полів з "кубічними" кореляційними функціями дає змогу доповнювати дані з достатньою точністю.***

К л ю ч о в і с л о в а : ***статистичне моделювання, "кубічна" кореляційна функція, спектральний розклад, кондиційність карт.***

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