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DETERMINING THE FOCAL MECHANISM OF AN EARTHQUAKE IN THE TRANSCARPATHIAN REGION OF UKRAINE

(Reviewed by the editorial board member G. Prodaivoda)

In the paper, the theory of seismic wave propagation in anisotropic media is presented, with the use of the matrix method when determining the mechanisms of local earthquakes. These issues are of importance in seismic studies in the Carpathian region where the number of seismic stations is insufficient.

The work is aimed at the development of a methodology for calculating displacements on the surface of anisotropic medium and determining the mechanisms of local earthquakes with the use of the graphic method. The purpose of the graphic method consists in making it possible to use seismic records at the stations with indistinct polarities of first P-wave motions. The ratio between the amplitudes of direct P- and S-waves is used as an auxiliary parameter.

The research results are illustrated with examples of using the records of 04.04.2013 event near Nyzhnye Selyshche (φ=48,1977, λ=23,4663, h=1,73 km, M_L=2). Comparative analysis is carried out of seismograms calculated with the matrix method and those recorded at a seismic station, which confirms the effectiveness of the methodology for determining the seismic source parameters. Based on the graphic method, spectral and geometric parameters of the seismic source have been obtained: seismic moment, radius of shear dislocation, slip area, mean fault slip, stress drop, energy and magnitude.

Scientific novelty of the work consists in developing a method of calculating the displacement field in anisotropic media with the use of the matrix method and in amending the graphic method so as to ensure determining the mechanisms of local earthquakes in the Carpathian region where the number of seismic stations is limited.

Practical application of the work is in determining the source parameters of local earthquakes, based on the developed approaches, which is of crucial importance in local seismological studies.

The version of the matrix method developed in this work for calculating seismic wave propagation in anisotropic media can be used for determining the tensor of seismic moment with the number of seismic stations being limited.

Introduction

Determining earthquake focal mechanisms in the Transcarpathian region of Ukraine is one of the most topical issues in local seismological investigations. As the level of local seismic activity is low, the number of reliable polarities of first motions at local seismic stations is very often insufficient to determine the mechanism in a traditional way, which necessitates the development of alternative methods and improvement of the traditional ones.

An approach is often used where nodal planes are plotted on a lower-hemisphere stereographic projection to best fit the polarities of first arrivals of *P* waves at the stations, the location of a station polarity on the projection depending on the station azimuth and take-off angle of the ray of first arrival connecting the source and the station.

These focal mechanisms are determined using a method that attempts to find the best fit to the direction of *P*-wave first motions observed at each station. For a double-couple source mechanism (or only shear motion on the fault plane), the compression first motions should lie only in the quadrant containing the tension axis, and the dilatation first motions should lie only in the quadrant containing the pressure axis. Accuracy of the focal mechanism solution depends on the input data: velocity model and coordinate of the hypocenter (they determine the take-off angle), quality of seismic records and sign inversion on the seismometer, so that "up" is "down" (they determine the entry wave character).

But sometimes there is not enough information about the first arrivals of *P* waves. Both information about fuzzy arrivals of *P*-waves (which can mean proximity to the nodal plane) and the value logarithm of the amplitude ratio of *S*-wave and *P*-wave amplitude at each station is important for the distribution of compression and tension by nodal lines in the quadrants.

However, it is appropriate to develop other methods for determining the parameters of an earthquake source. One of these methods is based on the expressions for displacement field on free surface of an anisotropic medium and spectra of real records from stations that recorded these events.

Using the Thomson – Haskell matrix method of constructing wave fields on the free surface is also feasible. A method has been developed here for mathematical modeling of elastic waves in medium consisting of homogeneous anisotropic layers with parallel boundaries.

At the boundaries between the layers, the condition of the hard contact is performed. A free surface is free of stress. Wave source is located inside an anisotropic layer at a certain depth $z=z_s$. The radiation condition is also fulfilled (the wave of the lower half space (n+1) does not return).

The solution is shown here for the direct problem when a point source represented by a randomly oriented force on an arbitrary boundary of a layered anisotropic medium is preset. A "wave propagator" is introduced in order to present the theory of the matrix propagator in a homogeneous anisotropic medium. The basic expressions for the stress-displacement field with using the matrix propagator and the radiation condition are obtained. In fact, the direct problem is reduced to the determination of the propagator $P(z,z_0)$.

To determine the earthquake source parameters, we use the spectra of real records and the basic expressions for the stress-displacement field with the matrix propagator.

Matrix method. Direct problem

We assume the usual linear relationship between stress τ_{ij} and strain e_{kl}

$$\tau_{ij} = c_{ijkl} \cdot e_{kl} = c_{ijkl} \frac{\partial u_l}{\partial x_k} \quad (1)$$

where $\vec{u} = (u_x, u_y, u_z)^T$ is displacement vector.

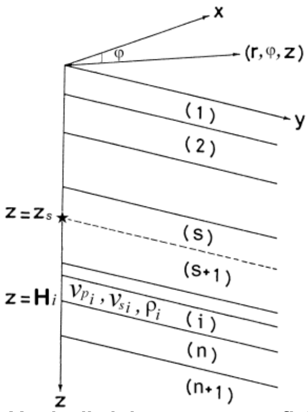

Figure 1. Vertically inhomogeneous field model

The equation of motion for an elastic homogeneous anisotropic medium, in the absence of body forces, is [*Fryer et al,* 1984]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_l}{\partial x_i \partial x_k}, \quad (2)$$

where ρ is the uniform mass density, and c_{ijkl} are the elements of the uniform elastic coefficient tensor which satisfy the symmetry conditions

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij},$$

so that only 21 independent constants are involved. The suffixes can take the values 1, 2, or 3, and the summation convention for repeated suffixes is assumed.

Taking the Fourier transform of (1) and (2), we obtain the matrix equation [6]

$$\frac{\partial \vec{b}}{\partial z} = j\omega A(z)\vec{b}(z), \quad (3)$$

where $\vec{b} = \begin{pmatrix} \vec{u} \\ \vec{\tau} \end{pmatrix}$ is the vector of displacements and scaled tractions, $\vec{\tau} = -\frac{1}{j\omega}(\tau_{xz}, \tau_{yz}, \tau_{zz})^T$. With the definition of $\vec{b}$ the system matrix A has the structure:

$$A = \begin{pmatrix} T & C \\ S & T^T \end{pmatrix};$$

where T, S and C are 3×3 sub matrices, C and S are symmetric.

For any vertically stratified medium, the differential system (3) can be solved subject to specified boundary conditions to obtain the response vector b at any desired depth. If the response at depth z_0 is $\vec{b}(z_0)$, the response at depth z is

$$\vec{b}(z) = P(z, z_0)\vec{b}(z_0) \quad (4)$$

where $P(z, z_0)$ is the stress-displacement propagator. The matrix propagator is defined as

$$P(z,z_0) = I + \int_{z_0}^{z} A(\xi_1)d\xi_1 + \int_{z_0}^{z} A(\xi_1)\int A(\xi_1)A(\xi_2)d\xi_2 d\xi_1 + \ldots, \quad (4^*)$$

where I is the 6 x 6 identity. If D is the local eigenvector matrix of A then

$$D^{-1}AD = \Lambda \quad (5)$$

where Λ is diagonal. The diagonal elements of Λ are the eigenvalues of A which are the vertical phase slownesses $q = p_z$. In general, we may write

$$\Lambda = diag(q_p^U, q_{s_1}^U, q_{s_2}^U, q_p^D, q_{s_1}^D, q_{s_2}^D), \quad (6)$$

where superscripts U and D denote upgoing and downgoing disturbances, the subscript P denotes quasi-P and S_1, S_2 denote the two types of quasi-S. For an isotropic medium $q^U = -q^D$, but for general anisotropy there is no such simple relationship between the vertical slownesses [9]. However, for our choice of Fourier transform and the definition of A in (3), it follows from the radiation condition that $Im(q^D)>0$ and $Im(q^U)<0$.

Given the eigenvector matrix D, we may define a wavevector $\vec{v}$ from the transformation

$$\vec{b} = D\vec{v}. \quad (6)$$

As in the isotropic case the elements of $\vec{v}$ may be identified with the amplitudes of upward and downward travelling plane waves,

$$\vec{v} = [v_u, v_D]^T = [\varphi_u, \psi_u, \chi_u, \varphi_D, \psi_D, \chi_D]^T \quad (7)$$

where φ, denotes qP amplitude and ψ, χ the two qS amplitudes. As before, U and D denote up and down.

If the elastic parameters are locally constant, then D is independent of z and substitution of (6) and (5) into (3) yields

$$D\frac{\partial \vec{v}}{\partial z} = j\omega AD\vec{v} \quad (8)$$

with the solution

$$\vec{v}(z) = e^{j\omega\Lambda(z-z_1)} \cdot \vec{v}(z_1) = Q(z, z_1) \cdot \vec{v}(z_1) \quad (9)$$

where z_1 is a reference depth. From (7) it is apparent that Q may be regarded as a 'wave propagator' since it is the solution to

$$\frac{\partial Q(z,z_1)}{\partial z} = j\omega\Lambda Q(z,z_1), \; Q(z_1,z_1) = I \quad (10)$$

We note from (6) that within the uniform layer, Q has the structure [6]

$$Q(z,z_1) = \begin{pmatrix} E_u & 0 \\ 0 & E_D \end{pmatrix} \quad (11)$$

with

$$E_u = diag\left[e^{j\omega(z-z_1)q_p^u}, e^{j\omega(z-z_1)q_{s_1}^u}, e^{j\omega(z-z_1)q_{s_2}^u}\right],$$

$$E_D = diag\left[e^{j\omega(z-z_1)q_p^D}, e^{j\omega(z-z_1)q_{s_1}^D}, e^{j\omega(z-z_1)q_{s_2}^D}\right] \quad (12)$$

Using (6) and (9), the stress-displacement vector at any level z within the uniform medium is

$$\vec{b}(z) = DQ(z,z_1)D^{-1}\vec{b}(z_1). \quad (13)$$

By comparison with (4), the desired propagator for the uniform interval is

$$P(z,z_1) = DQ(z,z_1)D^{-1} \quad (14)$$

To find this propagator, it is necessary to find the eigenvalues (vertical slownesses), the eigenvector matrix D, and its inverse D^{-1}. In the isotropic case these are known analytically, so the construction of the propagator is straightforward. In the anisotropic case, analytic solutions have been found only for simple symmetries, so in general, solutions can be found numerically.

The layered anisotropic medium, which consists of n homogeneous anisotropic layers on $(n+1)$ anisotropic half space (Figure 1), is considered. The matrix propagator (4*) can be represented by a "wave propagator" in each layer for an anisotropic layered medium. The source in the form of a stress-displacement discontinuity $\vec{F}=\vec{b}_{s+1}-\vec{b}_s$ is placed on the s-boundary (Fig. 1); it is easy to write the following matrix equation, using (13-14):

$$\vec{b}_{n+1}=P_{n,s}\vec{b}_{s+1}\Big|_{z=z_s},$$

$$v_{n+1}=D_{n+1}^{-1}D_nQ_nD_n^{-1}\cdots D_{s+1}Q_{s+1}D_{s+1}^{-1}\cdot\vec{b}_{s+1}\Big|_{z=z_s},$$

$$\vec{b}_s\Big|_{z=z_s}=P_{s,s-1}P_{s-1,s-2}\cdots P_{2,1}P_{1,0}\cdot\vec{b}_0= \\ =D_sQ_sD_s^{-1}\cdots D_1Q_1D_1^{-1}\cdot\vec{b}_0,$$

$$v_{n+1}=D_nQ_nD_n^{-1}\cdots D_{s+1}Q_{s+1}D_{s+1}^{-1}\cdot(\vec{b}_s+\vec{F})=G^{n+1,s+1}\cdot(G_{s,1}\vec{b}_0+\vec{F})=, \\ G^{n+1,s+1}=G_{s,,1}\vec{b}_0+G^{n+1,s+1}\cdot\vec{F}=G\vec{b}_0+G^{n+1,s+1}\cdot\vec{F}$$

where

$$G=D_{n+1}^{-1}D_nQ_nD_n^{-1}\cdots D_{s+1}Q_{s+1}D_{s+1}^{-1}\cdots D_2^{-1}D_1Q_1D_1^{-1}$$

– characteristic matrix of a layered anisotropic medium.

$$\vec{v}_{n+1}=G\vec{b}_0+G\cdot G_{s,1}^{-1}\cdot\vec{F}=G(\vec{b}_0+G_{s,1}^{-1}\cdot\vec{F})=G(\vec{b}_0+\vec{\tilde{F}}), \quad (15)$$

where $\vec{\tilde{F}}=G_{s,1}^{-1}\cdot\vec{F}$, $G=G^{n+1,s+1}\cdot G_{s,1}$.

Using (15) and the radiation condition (with a half-wave (*n* +1) not returned), and also the fact that the tension on the free surface equals zero, we obtain a system of equations:

$$\begin{pmatrix}0\\0\\0\\v_D^P\\v_D^{S_1}\\v_D^{S_2}\end{pmatrix}=\begin{pmatrix}G_{11}&G_{12}&G_{13}&G_{14}&G_{15}&G_{16}\\G_{21}&G_{22}&G_{23}&G_{24}&G_{25}&G_{26}\\G_{31}&G_{32}&G_{33}&G_{34}&G_{35}&G_{36}\\G_{41}&G_{42}&G_{43}&G_{44}&G_{45}&G_{46}\\G_{51}&G_{52}&G_{53}&G_{54}&G_{55}&G_{56}\\G_{61}&G_{62}&G_{63}&G_{64}&G_{65}&G_{66}\end{pmatrix}\begin{pmatrix}u_x^{(0)}+\tilde{F}_1\\u_y^{(0)}+\tilde{F}_2\\u_z^{(0)}+\tilde{F}_3\\\tilde{F}_4\\\tilde{F}_5\\\tilde{F}_6\end{pmatrix} \quad (16)$$

or

$$\begin{cases}G_{11}u_x^{(0)}+G_{12}u_y^{(0)}+G_{13}u_z^{(0)}=-(G_{11}\tilde{F}_1+G_{12}\tilde{F}_2+G_{13}\tilde{F}_3+G_{14}\tilde{F}_4+G_{15}\tilde{F}_5+G_{16}\tilde{F}_6)\\G_{21}u_x^{(0)}+G_{22}u_y^{(0)}+G_{23}u_z^{(0)}=-(G_{21}\tilde{F}_1+G_{22}\tilde{F}_2+G_{23}\tilde{F}_3+G_{24}\tilde{F}_4+G_{25}\tilde{F}_5+G_{26}\tilde{F}_6)\\G_{31}u_x^{(0)}+G_{32}u_y^{(0)}+G_{33}u_z^{(0)}=-(G_{31}\tilde{F}_1+G_{32}\tilde{F}_2+G_{33}\tilde{F}_3+G_{34}\tilde{F}_4+G_{35}\tilde{F}_5+G_{36}\tilde{F}_6)\end{cases} \quad (17)$$

As a result, the displacement field of the free surface of the anisotropic medium is in the spectral domain as:

$$\vec{u}=\begin{pmatrix}u_x^0\\u_y^0\\u_z^0\end{pmatrix}=(G^{13})^{-1}\cdot\vec{y} \quad (18)$$

where $G^{13}=\begin{pmatrix}G_{11}&G_{12}&G_{13}\\G_{21}&G_{22}&G_{23}\\G_{31}&G_{32}&G_{33}\end{pmatrix}$, $\vec{y}=\begin{pmatrix}a\\b\\c\end{pmatrix}$,

$$a=-(G_{11}\tilde{F}_1+G_{12}\tilde{F}_2+G_{13}\tilde{F}_3+G_{14}\tilde{F}_4+G_{15}\tilde{F}_5+G_{16}\tilde{F}_6),$$
$$b=-(G_{21}\tilde{F}_1+G_{22}\tilde{F}_2+G_{23}\tilde{F}_3+G_{24}\tilde{F}_4+G_{25}\tilde{F}_5+G_{26}\tilde{F}_6),$$
$$c=-(G_{31}\tilde{F}_1+G_{32}\tilde{F}_2+G_{33}\tilde{F}_3+G_{34}\tilde{F}_4+G_{35}\tilde{F}_5+G_{36}\tilde{F}_6).$$

Using (18) and the three-dimensional Fourier transform, we obtain a direct problem solution for the displacement field of the free surface of an anisotropic medium in the time domain as:

$$\vec{u}(x,y,z_R,t)= \\ =\frac{1}{8\pi^3}\iiint_{-\infty}\omega^2\vec{u}(p_x,p_y,z_R,\omega)e^{j\omega(t-p_xx-p_yy)}dp_xdp_yd\omega,$$

where z_R – epicentral distance, p_x, p_y – horizontal slowness.

The stress-displacement discontinuity is determined via the seismic in matrix form [7]:

$$\vec{F}=\begin{pmatrix}-c_{55}^{-1}M_{xz}\\-c_{44}^{-1}M_{yz}\\-c_{33}^{-1}M_{zz}\\p_x(M_{xx}-c_{13}c_{33}^{-1}M_{zz})+p_yM_{xy}\\p_xM_{yx}+p_y(M_{yy}-c_{23}c_{33}^{-1}M_{zz})\\p_x(M_{zx}-M_{xz})+p_y(M_{zy}-M_{yz})\end{pmatrix}\delta(z-z_z) \quad (19)$$

where M_{xx}, M_{yy}, M_{zz}, M_{xz}, M_{yz}, M_{yx}, M_{xy}, M_{zy}, M_{zx} – components of the seismic moment tensor, and c_{13}, c_{23}, c_{33}, c_{44}, c_{55} – components of the stiffness matrix.

Matrix method. Inverse problem

We can write the stress-displacement discontinuity $\vec{F}$ for weak seismic events as:

$$\vec{F}=\begin{pmatrix}F_1\\F_2\\F_3\\0\\0\\0\end{pmatrix}. \quad (20)$$

Using (16, 19, 20), we obtain a system of equations, which has a unique solution:

$$\begin{cases}G_{11}u_x^{(0)}+G_{12}u_y^{(0)}+G_{13}u_z^{(0)}=-(G_{11}\tilde{F}_1+G_{12}\tilde{F}_2+G_{13}\tilde{F}_3)\\G_{21}u_x^{(0)}+G_{22}u_y^{(0)}+G_{23}u_z^{(0)}=-(G_{21}\tilde{F}_1+G_{22}\tilde{F}_2+G_{23}\tilde{F}_3)\\G_{31}u_x^{(0)}+G_{32}u_y^{(0)}+G_{33}u_z^{(0)}=-(G_{31}\tilde{F}_1+G_{32}\tilde{F}_2+G_{33}\tilde{F}_3)\end{cases}. \quad (21)$$

$$\vec{F}=G_{s,1}\cdot\vec{\tilde{F}}, \quad (22)$$

where $G_{s,1}$ is the characteristic matrix of the source.

Using (21, 22), we find explicitly the stress-displacement vector $\vec{F}$. From (19) equations for the determination of the seismic moment tensor components are derived:

$$M_{xz}=M_{zx}=-F_1\cdot c_{55} \\ M_{yz}=M_{zy}=-F_2\cdot c_{44} \quad (23) \\ M_{zz}=-F_3\cdot c_{33}$$

To determine the angles of orientation of the plane of rupture (φ_s, δ, λ), we use the trigonometric system of equations which represent the known components of seismic moment tensor via the angles of orientation of the plane of rupture [1]:

$$M_{xz} = -M_0 \cdot (\cos\delta \cdot \cos\lambda \cdot \cos\phi_s + \cos 2\delta \cdot \cos\lambda \cdot \cos\phi_s)$$
$$M_{yz} = -M_0 \cdot (\cos\delta \cdot \cos\lambda \cdot \cos\phi_s - \cos 2\delta \cdot \sin\lambda \cdot \cos\phi_s)\ , \quad (24)$$
$$M_{zz} = M_0 \sin 2\delta \cdot \sin\lambda$$

where M_0 – seismic moment determined from the spectrum of seismograms. The found angles of orientation of the plane of rupture (φ_s, δ, λ) are substituted in the following equation to find the total seismic moment tensor [1]:

$$M_{xx} = -M_0 \cdot (\sin\delta \cdot \cos\lambda \cdot \sin 2\phi_s + \sin 2\delta \cdot \sin\lambda \cdot \sin^2\phi_s)$$
$$M_{xy} = M_{yx} = M_0 \cdot (\sin\delta \cdot \cos\lambda \cdot \cos 2\phi_s + \frac{1}{2}\sin 2\delta \cdot \sin\lambda \cdot \sin 2\phi_s) \quad (25)$$
$$M_{yy} = M_0 \cdot (\sin\delta \cdot \cos\lambda \cdot \sin 2\phi_s - \sin 2\delta \cdot \sin\lambda \cdot \cos^2\phi_s)$$

As a result, we write the seismic moment tensor using the symmetry condition:

$$M = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{xy} & M_{yy} & M_{yz} \\ M_{xz} & M_{yz} & M_{zz} \end{pmatrix}. \quad (26)$$

Determining the focal mechanism by graphical method

Today, the focal mechanism solution for earthquakes in a region of low seismic activity is a topical issue. It is of crucial importance for the Transcarpathian region of Ukraine, where the number of stations is limited and seismic activity is low. It is impossible to determine a focal mechanism with software packages.

It is proposed to determine the focal mechanism by applying the traditional graphical method based on the first arrival P-waves [10] using the information about fuzzy first motion [5] and the *S*/*P* amplitude ratio [8].

To test the graphical method, an event dated 04.04.2013 21:15:14.36 is considered (φ=48.1977, λ=23.4663, h=1.73 km) near the village Nyzhnye Selyshche. This event was recorded by 9 stations (Figure 2)

Figure 2. Map of seismic stations in the Transcarpathian region and the specified location near to the epicenter of events near Nyzhnye Selyshche village

The polarities of first motion P-waves were defined from complete records seismograms taking into account the possible inversion of the sign on the *z*-component. A logarithm of the amplitude ratio S/P is calculated using data from the three components seismic records of this event at each station [8]. Input data for the azimuth and take-off angle are calculated by software packages for this event (Table 1).

Table 1

Input data for the focal mechanism solution

Stations	First arrival	Azimuth,˚	Take-off angle,˚	*lg* As/Ap
NSLU	+ Pg	269	-53	-
KORU	-Pg	260	31	0.43
MEZ	-Pg	6	31	0.084
BRIU	-Pn	295	42	0.65
TRSU	-Pn	253	42	0.57
BERU	xPn	274	42	2.64
MUKU	-Pn	297	42	0.71
UZH	-Pn	299	45	0.88
KSV	-Pn	83	45	1.05

The graphical method is used to determine the focal mechanism for this event according to the input data [10]. The data about first motion P-wave is plotted using a lower-hemisphere stereographic projection. The point-projection rays from the source to the station were applied on the stereonet. The red points are the points of compression (where the *P*-wave first motion recorded was up), the blue points are the points of dilatation (where the *P*-wave first motion recorded was down), and black points are the points of fuzzy first motion. Several averaged versions of the nodal plane locations are determined out of all the possible alternative versions for this event (Figure 3).

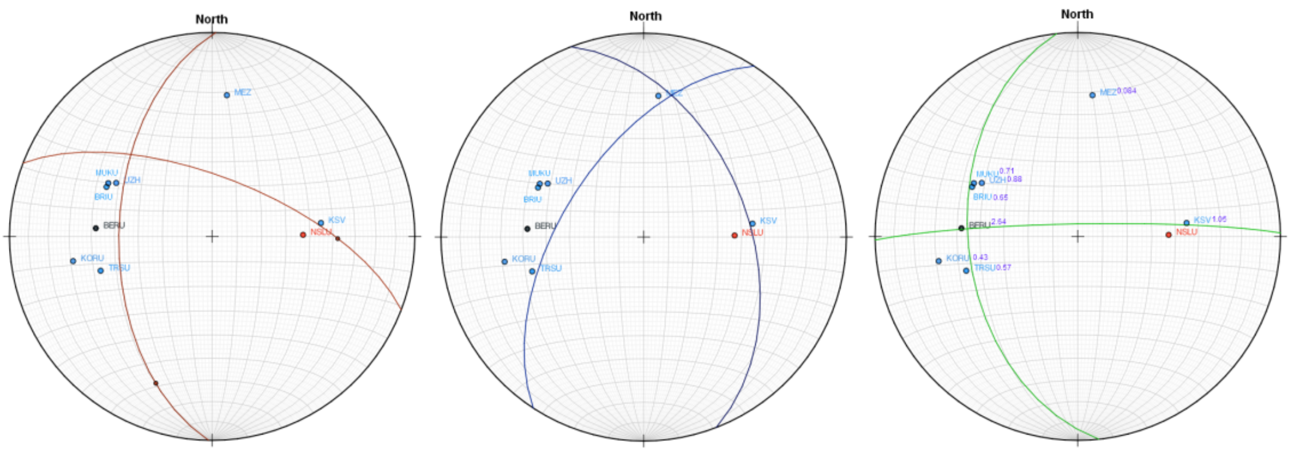

Figure 3. Averaged versions of the nodal planes locations for event 04.04.2013 21:15:14.36 (φ=48.1977, λ=23.4663, h=1.73 km) near the village Nyzhnye Selyshche

The determining criteria for the most suitable locations of the nodal planes for the 4.04.2013 earthquake were: fuzzy first motion on the station BERU, a small logarithm value of amplitude ratio S/P at the station MEZ indicates a location projection station in the middle of the quadrant, and the logarithm value of the amplitude ratio S/P at the station KSV, which is larger than 1, indicates the proximity to the nodal line. Consequently, the focal mechanism for event 04.04.2013 21:15:14.36 (φ=48.1977, λ=23.4663, h=1.73 km) near the village Nyzhnye Selyshche is represented by diagram (Figure 4) with parameters in Table 2.

Table 2

Parameters of the focal mechanism for the event 04.04.2013 21:15:14.36 (φ=48.1977, λ=23.4663, h=1.73 km) near the village Nyzhnye Selyshche

Plane1			Plane2			P		T		N	
Strike (φ_s)	Dip (δ)	Slip (λ)	Strike (φ_s)	Dip (δ)	Slip (λ)	Azm	Plunge	Azm	Plunge	Azm	Plunge
174°	45°	173°	269°	85°	45°	33°	27°	142°	34°	274°	44°

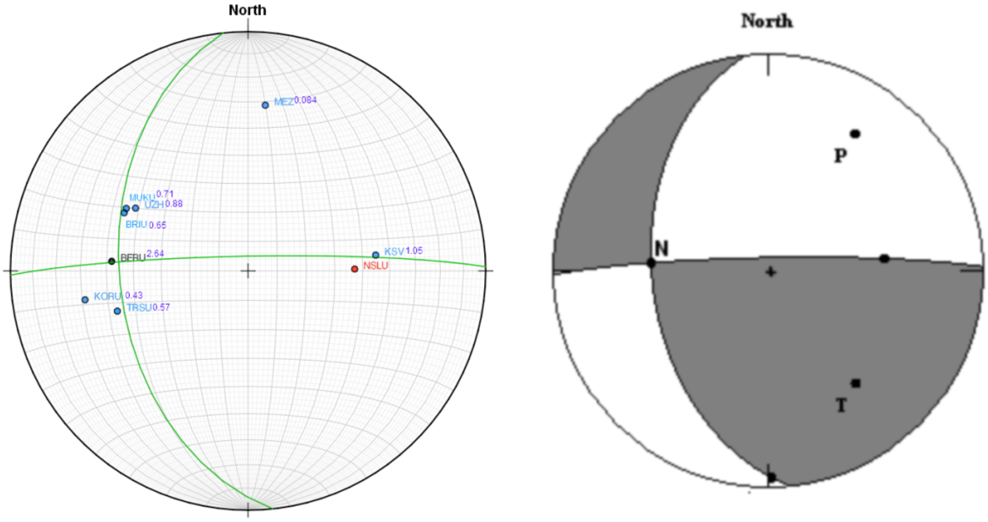

Figure 4. The focal mechanism determined by graphic method for the event04.04.2013 21:15:14.36 (φ=48.1977, λ=23.4663, h=1.73 km) near the village Nyzhnye Selyshche

Seismic moment and other spectral parameters are computed by (27-33) [2] for each station and the average values of these parameters are represented in Table 3.

The seismic moment is computed according to:

$$M_0 = 4\pi r v_p^3 \rho u_0 / (\Theta S_a), \quad (27)$$

where r – is hypocentral distance, v_p - *P*-wave velocity, *ρ* – density, u_0 – low-frequency level (plateau) of the displacement spectrum, *Θ* – average radiation pattern and S_a – surface amplification for *P* waves.

The radius of shear dislocation R is computed from the relationship:

$$R = \frac{3.36 v_p}{2\sqrt{3}\pi f_c}, \quad (28)$$

where f_c - is the corner frequency of the *P* wave. The size of the circular rupture plane is computed as:

$$A = \pi R^2. \quad (29)$$

The average source dislocation is according to

$$\overline{D} = M_0 / \mu A, \quad (30)$$

where the shear modulus is computed by

$$\mu = v_p^2 \rho / 3. \quad (31)$$

The stress drop, seismic energy and magnitude ML are computed according to:

$$\Delta\sigma = 7M_0 / 16R^3, \quad (31)$$

$$E_s = M_0 \cdot 1.6 \cdot 10^{-5}, \quad (32)$$

$$ML = (lg\, E_s - 4)/1.8. \quad (33)$$

With the seismic moment and the parameters of the focal mechanism, the moment tensor M is defined from (24, 25) [1]:

$$M = \begin{pmatrix} -11.42494 & -54.24707 & -54.15575 \\ -54.24707 & 1.96935 & 5.69199 \\ -54.15575 & 5.69199 & 9.45559 \end{pmatrix} \cdot 10^{11} \quad (34)$$

Approbation of the inverse problem

The inverse problem is solved for the event, which took place near the village Nyzhnje Selyshche.

The orientation angles of the fault plane are determined by the graphic method (φ_s = 174°, *δ* = 45 °, *λ* = 173°) and the focal mechanism is determined for the event. The seismic tensor (27) is obtained by substituting the orientation angles of the fault plane and the magnitude of seismic moment in (24, 25).

The real seismic records at station Mezhyhirya is used for the inverse problem. The earthquake source is located in the first layer. Therefore, a two-layered anisotropic model of medium (with TI symmetry) is selected, whose parameters are given in Table 4.

The orientation angles of the fault plane (φ_s = 177°, *δ* = 45°, *λ* = 175°) are obtained as a result of the inverse problem solving using spectrum of real seismic record and the velocity model (Table 4). The seismic tensor is obtained by substituting the orientation angles of the fault plane and the magnitude of seismic moment in (24, 25):

$$M = \begin{pmatrix} -5.73144 & -54.70825 & -54.57932 \\ -54.70825 & -1.03079 & 2.86038 \\ -54.57932 & 2.86038 & 6.76224 \end{pmatrix} \cdot 10^{11} \quad (35)$$

The focal mechanism is shown in Figure 4 which is based on the seismic moment tensor (35).

Table 3

Spectral parameters for the event 04.04.2013 21:15:14.36 (φ=48.1977, λ=23.4663, h=1.73 km) near the village Nyzhnye Selyshche

M_0, Nm	f_{cp}, Hz	R, m	A, m^2	$\overline{D}$, m	$\Delta\sigma$, MPa	E_s, J	ML
6.68784*10^{12}	6.81	213.191	1.4271*10^{5}	2.85*10^{-3}	0.302	1.07*10^{8}	2.22

Table 4

Velocity model for seismic station Mezhyhirya

№	c_{11}, GPa	c_{13}, GPa	c_{33}, GPa	c_{44}, GPa	c_{66}, GPa	ρ, kg/m^3	h, m
1	81.12	25.342	84.397	37.036	28.127	3000	2400
2	100.38	33.464	100.38	33.458	33.458	3367	6600

Table 5

Velocity model for seismic station Korolevo

№	c_{11}, GPa	c_{13}, GPa	c_{33}, GPa	c_{44}, GPa	c_{66}, GPa	ρ, kg/m^3	h, m
1	76.81	24.57	76.71	24.36	24,26	3000	2400
2	100.38	33.464	100.38	33.458	33,458	3367	6600

where c_{11}, c_{13}, c_{33}, c_{44}, c_{66} – components of the stiffness matrix, ρ – density, h – thickness of layer.

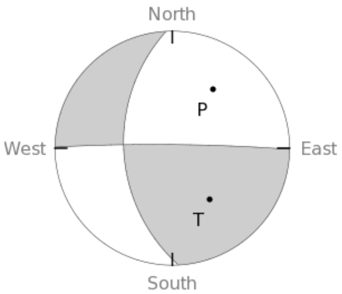

Figure 5. The focal mechanism determined by the proposed method for the event which took place near the village Nyzhnje Selyshche (φ_s = 177°, δ = 45 °, λ = 175°)

The focal mechanisms of the earthquake built by two different methods (Figure 4–5) are actually identical, which confirms the correctness and accuracy of the matrix method.

The synthetic seismograms are constructed for the earthquake's focal mechanism (Figure 5) and the velocity models (Table 4-5) to confirm the inverse problem solutions. A comparative analysis is done of synthetic seismograms and real records at the stations Mezhyhirya and Korolevo, which are filtered in the frequency range from f_0 = 0.1Hz to f_{max} = 5Hz (Figure 6-7).Synthetic seismograms are built for the obtained seismic tensor (35) and the velocity model (Table 4-5).

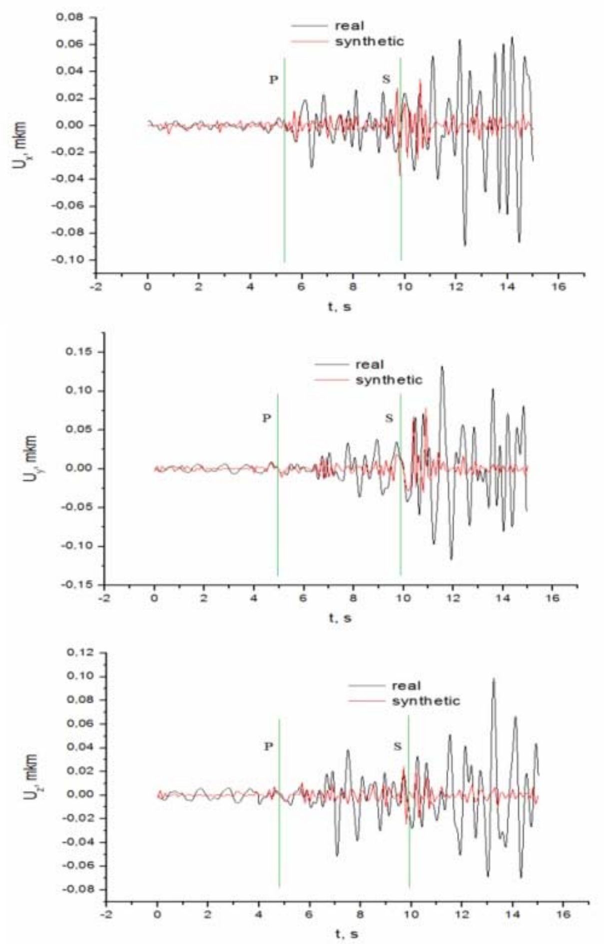

Figure 6. Comparison of synthetic seismograms with real seismic record from station Mezhyhirya

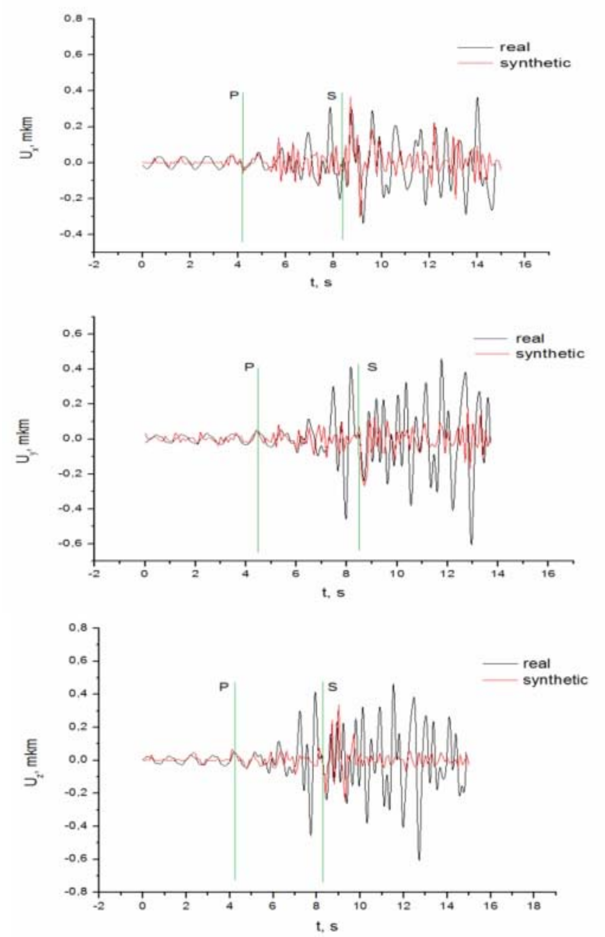

Figure 7. Comparison of synthetic seismograms with real seismic record from station Korolevo

Conclusion

Comparative analysis of waveforms confirms the feasibility of using the matrix method for solving seismology problems with earthquakes sources being distributed in time. Similarity of the focal mechanisms obtained by two different methods confirms the correctness of the solutions for this event.

More accurate results in determining the earthquake focal mechanisms are obtained when using the spectrum data from stations that are located at a smaller epicentral distance. The best results were obtained for those stations where records have a lower noise level. Choosing the velocity model is essential for determining earthquake focal mechanisms.

We can conclude that the graphical method is suitable for determining focal mechanisms for earthquakes in the Carpathian region of Ukraine.

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ВИЗНАЧЕННЯ ФОКАЛЬНОГО МЕХАНІЗМУ ЗЕМЛЕТРУСУ В ЗАКАРПАТТІ

У роботі представлено теорію поширення сейсмічних хвиль в анізотропному середовищі з використанням матричного методу Томсона-Хаскела, а також визначення механізмів вогнищ місцевих землетрусів. Такі задачі є надзвичайно актуальні для вивчення сейсмічності Закарпаття через обмежену кількість сейсмічних станцій.

Метою роботи є розроблення методики для побудови поля переміщень на вільній поверхні анізотропного середовища і визначення механізмів вогнищ місцевих землетрусів графічним методом. Суть графічного методу полягає у використанні сейсмічних записів на станціях з неточним вступом прямих Р-хвиль. Як допоміжний параметр у роботі використано відношення амплітуд прямих Р і S хвиль.

Результати запропонованих підходів показано на прикладі використання записів події 04.0.2013 р. біля с. Н. Селище (φ=48.1977; λ=23.4663; h=1.73 км, ML=2). Зокрема, представлено порівняльний аналіз сейсмограм, отриманих матричним методом із реальними записами, що підтверджує використання методики для визначення параметрів джерела. На основі графічного методу для визначення механізмів вогнищ місцевих землетрусів отримано спектральні та геометричні параметри джерела: сейсмічний момент, радіус зсувної дислокації, площу розриву, середню посувку по розриву, спад напруги, енергію та магнітуду.

Наукова новизна роботи полягає у розроблені методики визначення поля переміщень у випадку анізотропного середовища з використанням матричного методу, а також розвитку графічного методу для побудови механізмів вогнищ землетрусів Закарпаття у випадку обмеженої кількості станцій.

Практична значимість роботи полягає в тому, що на основі розроблених підходів є можливість визначення параметрів вогнищ місцевих землетрусів, що є важливим для вивчення сейсмічності регіону. Розроблена модифікація матричного методу для поширення сейсмічних хвиль в анізотропних середовищах може бути використана для визначення тензора сейсмічного моменту у випадку обмеженої кількості станцій.

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ОПРЕДЕЛЕНИЕ ФОКАЛЬНОГО МЕХАНИЗМА ЗЕМЛЕТРЯСЕНИЯ В ЗАКАРПАТЬЕ

В работе представлено теорию распространения сейсмических волн в анизотропной среде с использованием матричного метода Томсона – Хаскела, а также определения механизмов очагов местных землетрясений. Такие задачи чрезвычайно актуальны для изучения сейсмичности Закарпатья из-за ограниченного количества сейсмических станций.

Целью работы является разработка методики для построения поля перемещений на свободной поверхности анизотропной среды и определения механизмов очагов местных землетрясений графическим методом. Суть графического метода заключается в использовании сейсмических записей на станциях с неточным вступлением прямых Р-волн. Как вспомогательный параметр в работе использовано отношение амплитуд прямых Р и S волн.

Результаты предложенных подходов показано на примере использования записей события 04.0.2013 г. в районе с. Н. Селище (φ = 48.1977; λ = 23.4663; h = 1.73 км, ML = 2). В частности, представлен сравнительный анализ сейсмограмм, полученных матричным методом, с реальными записями, подтверждающий возможность использования методики для определения параметров источника. На основе графического метода для определения механизмов очагов местных землетрясений получены спектральные и геометрические параметры источника: сейсмический момент, радиус сдвиговой дислокации, площадь разрыва, среднюю подвижку по разрыву, сброс напряжения, энергию и магнитуду.

Научная новизна работы заключается в разработанной методике определения поля перемещений в случае анизотропной среды с использованием матричного метода, а также развития графического метода для построения механизмов очагов землетрясений Закарпатья в случае ограниченного количества станций.

Практическая значимость работы заключается в том, что на основе разработанных подходов является возможность определения параметров очагов местных землетрясений, что важно для изучения сейсмичности региона. Разработанная модификация матричного метода для распространения сейсмических волн в анизотропных средах может быть использована для определения тензора сейсмического момента в случае ограниченного количества станций.