ГЕОЛОГІЧНА ІНФОРМАТИКА

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Z. Vyzhva, Dr. Sci. (Phys.-Math.), Prof.,
E-mail: zoya_vyzhva@ukr.net;
V. Demidov, PhD (Phys.-Math.), Assoc. Prof.,
E-mail: fondad@ukr.net;
Taras Shevchenko National University of Kyiv,
90 Vasylkivska Str., Kyiv, 03022, Ukraine;
A. Vyzhva, PhD (Phys.-Math.), Senior Researcher,
E-mail: motomustanger@ukr.net;
SE "Naykanafto-gaz", Kyiv, Ukraine

THE STATISTICAL SIMULATION OF DATASET IN 3D AREA WITH SPHERICAL CORRELATION FUNCTION ON RIVNE NPP EXAMPLE

(Представлено членом редакційної колегії д-ром фіз.-мат. наук, проф. Б.П. Масловим)

Due to the increasing number of natural and technogenic disasters, the development of geological environment monitoring system is actual in using modern mathematical tools and information technology. The local monitoring of potentially dangerous objects is an important part of the overall environment monitoring system.

The complex geophysical research was conducted on Rivne NPP area. The monitoring observations radioisotope study of soil density and humidity near the perimeter of buildings is of the greatest interest among these.

In this case a problem occurred to supplement simulated data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells.

This problem was solved in this work by statistical simulation method that provides the ability to display values (the random field of a research object in 3D area) in any point of the monitoring area. The chalk strata averaged density at the industrial area was simulated using the built model and the involvement optimal in the mean square sense spherical correlation function.

In this paper, the method is used and the model and procedure were developed with enough adequate data forsphericalcorrelation function.

The model and algorithm were developed and examples of karst-suffusion phenomena statistical simulation were given in the problem of density chalk strata monitoring at the Rivne NPP area. The statistical model of averaged density chalk strata distribution was built in 3D area and statistical simulation algorithm was developed using spherical correlation functionbased on spectral decomposition. The research subject realizations were obtained with required detail and regularity at the observation grid based on the developed software. Statistical analysis of the numerical simulation results was done and tested for its adequacy.

Keywords: Statistical simulation, spherical correlation function, spectral decomposition, conditional maps.

Introduction. Due to the increasing frequentative number of dangerous natural and technogenic disasters, the development of geological environment monitoring system is actual using modern mathematical tools and information technology. The regular local monitoring of potentially dangerous objects is an important part of the overall environment monitoring system. When the monitoring of such objects many actual problems were raised, for example, such as the lack of some data in the database, or insufficient quantity or necessity to supplement the database without conducting additional research.

Theoretical aspects of capacity use of the statistical simulation to solving different problems in the work of Geophysics considered in (*Yadrenko*, *1983; Vyzhva*, *2003, 2011*). Practical testing on real density chalky strata data on the Rivne NPP territory was carried out for the 3D area – in the following works (*Vyzhva et al., 2013, 2014, b*), by using Bessel correlation function and Cauchy correlation function.

In this paper, the statistical simulation method for random field in 3D area is proposed to use with the model and procedure involving enough adequate in the mean square sense data spherical correlation function (*Chiles et al., 2012*). This problem was considered also in paper (*Vyzhva et al., 2019*).

Note, that methods of 3D random fields statistical simulation used in geosciences problems was developed by the scientists: Mantoglov A., Wilson John L. (1981), Chiles J.P., Delfiner P. (2012), Wackernagel H. (2003), Prigarin S.M. (2005), Emery X. (2006) and other.

The problem of karst-suffusion phenomena monitoring at Rivne NPP 3D area.

The complex geophysical research was conducted on Rivne NPP 3D area during many years. The radioisotope study of soil density and humidity near the perimeter of constructed buildings is of the greatest interest among these monitoring observations. The soil density was determined by gamma-gamma well logging, soil humidity was determined also by neutron-neutron logging.

In this case (*Vyzhva et al., 2013*) a problem occurred to supplement adequately simulated data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells. Schematic representation of the measurement results at the Rivne NPP object that was investigated, and the well locations are shown on Fig. 1. These data are obviously not enough in detail to represent the overall picture of the chalk strata, where due to the aggressive water action the karst-suffusion processes were significantly intensified.

This noted problem was solved in folloving works: (*Vyzhva et al., 2013, 2014, a, 2014, b, 2019*) by statistical simulation method that provides the ability to display values (random field in 3D space) in any point of the monitoring area. The chalk strata averaged density at the industrial area was simulated using the built 3D model and the involvement of the Bessel type correlation function (*Vyzhva et al., 2013)* and Cauchy correlation function (*Vyzhva et al., 2014, b*).

This paper continues further development of 3D statistical simulation methods, involvingoptimal in the mean square sense spherical correlation function that is well-known in geostatisticworks (*Chiles et al., 2012*). This operation was done for data array of density chalk strata in 1984–2002s for 29 wells at Rivne NPP industrial area and depths are 28 m, 29 m and 30 m below the surface.

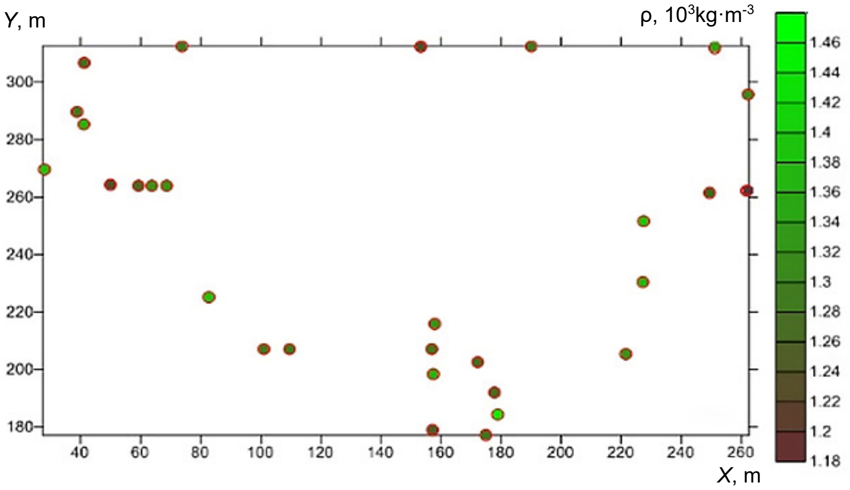


Fig. 1. Observation points and chalk strata averaged density at industrial area of Rivne NPP

The method of solving the problem. The statistical simulation of density chalky strata datawas performed at three levels (28, 29, 30 m from the surface)in this paper.While constructing graphs of density chalky strata data at the Rivne NPP object for each specified account, we noticed that it is expedient to distinguish deterministic and random components. Deterministic component can be selected as function by the method of approaching the minimum curve (separation of the so-called trend).The difference between the map of input density values and the trend is a realization of homogeneous isotropic random field in the most cases.

Input data on the each of three level from the surface is a realization of random field in 3D space $\eta(x,y,z_i), i=1,2,3;$ $z_1=28m, z_2=29m, z_3=30m.$ $\eta(x,y,z_i)=\eta_i(r,\theta,\varphi)$ (r,θ,φ) − spherical coordinates, *i* − level φnumbers. The trend $f_i(r,\theta,\varphi)$ and the stationary random component $\xi_i(r,\theta,\phi)$ (frequently homogeneous isotropic random fieldin 3D space, so-called "noise") were selected for each level:

$$\eta_i(r,\ \theta,\varphi)=f_i(r,\theta,\varphi)+\xi_i(r,\theta,\varphi), i=1,2,3.$$

The final modelling stage was the imposing array of random field realizations $\xi_i(r,\theta,\varphi), i=1,2,3$, what we got by statistical simulation in add points from 3D observations area on the approximation of real data. As a result, we received more detailed implementation for the chalk layer density data in the selected 3D area at Rivne NPP.

We use the method of statistical simulation of homogenous isotropic random fields in 3D area for the solving arising problem, which is based on their spectral decomposition (*Vyzhva*, *2003*). By means of realizations of obtained values, this technique allows to find the perfect image of these random fields in the whole observation area at the Rivne NPP.

It is necessary to make the statistical analysis of data to build the model and procedure of chalky stratadensity simulation at observation 3D area. If the verified 3D data has distribution density with approximately Gaussian type, then procedure can be used, which is developed in (*Vyzhva et al., 2013; Vyzhva, 2011*), to generate on the computer realizations of the simulated data by means of standard normal random variable sequences.

At first the distribution of chalky stratadensity data at the Rivne NPP is determined. The preliminary statistical analysis of 3D data shows that the distribution histogram of chalky strata density at the Rivne industrial area (29 boreholes) approximately has Gaussian distribution (Fig. 2).

The use of authors' techniques of statistical simulation implies preliminary statistical data processing to determine its statistical characteristics: the mathematical expectation and the correlation function model. If the hypothesis of Gaussian distribution of the investigated data field is confirmed, then the mathematical expectation and the correlation function completely define this random field in 3D area and give us the opportunity to build the adequate statistical model for data field, which is based on the spectral decomposition. The principles of constructing the models and procedures for the spherical correlation function were considered in work (*Vyzhva et al., 2019*) and described below.

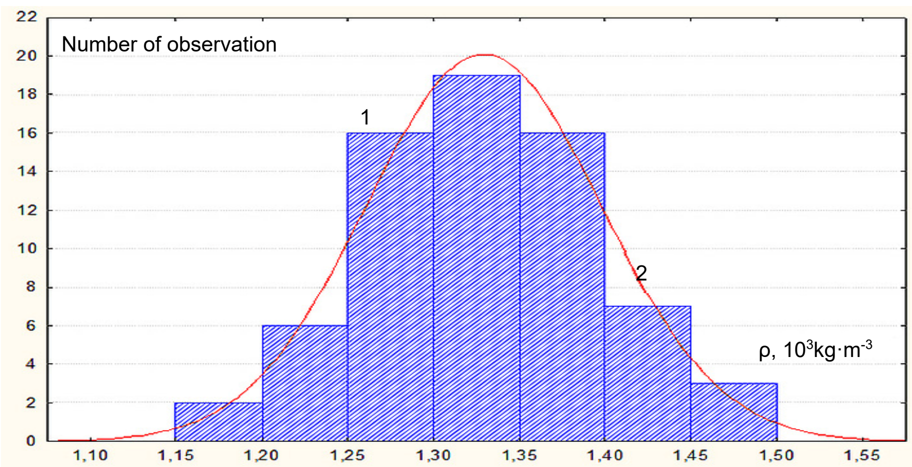


Fig. 2. The preliminary statistical analysis of 3D data at the Rivne industrial area

Then statistical models were chosen for the data correlation function $B(\rho)$ (ρ – the distance between vectors $x, y \in R^3$ ($x = (r_1, \theta_1, \varphi_1)$, $y = (r_2, \theta_2, \varphi_2)$) for distribution of chalky strata density in the 3D observation area. This function is defined by comparing the mean square approximation of the empirical and theoretical variograms of chalky stratadensity data. As a result, the input data was most adequately described by means of 3 types of correlation functions:the Bessel correlation function (1) at the value of parameter $c = 5$; the Cauchy correlation function (2) at the value of parameter $a = 1$ and the spherical correlation function (3) at the value of parameter $a \approx 1{,}25{*}10^{-2}$.

$$B(\rho) = \sqrt{\frac{\pi}{2cr}} J_{\frac{1}{2}}(c\rho),\ c = 5, \qquad (1)$$

where $J_k(x)$ is the Bessel function of the first kind of order $k = 1/2$,

$$B(\rho) = \frac{a^4}{\left(a^2 + \rho^2\right)^2},\ a = 1, \qquad (2)$$

$$B(\rho) = \begin{cases} 1 - \frac{3}{2}\frac{\rho}{a} + \frac{1}{2}\left(\frac{\rho}{a}\right)^3, & \rho \le a; \\ 0, & \rho > a. \end{cases} \qquad (3)$$

The graphic representations of the spherical function at value of parameter $a = 2$ are presented in Fig. 3

Variograms of input chalky strata density 3D data at the Rivne NPP was built by using the *R* software and geoR package. They corresponding to the: Bessel (1) correlation function (the mean square approximation is 0,0008599) and Cauchy (2) correlation function (the mean square approximation is 0,002816). Variograms plots were presented at Fig. 4, a, that according to Bessel type of correlation function, and at Fig. 4, b, that according to Cauchy type of correlation function for the random component of investigation 3D data.

The built variogram (Fig. 5) of input chalky strata density 3D data has the best approximation by theoretical variogram which is connected to the spherical correlation function (7) with parameter $a \approx 1{,}25{*}10^{-2}$ (a mean square deviation is $1{,}42{*}10^{-3}$).

We have, that the built variogram of implementations on the study area (Fig. 6) has enouph adequate approximation by theoretical variogram which is connected to the spherical correlation function (a mean square deviation is $4{,}8{*}10^{-4}$).

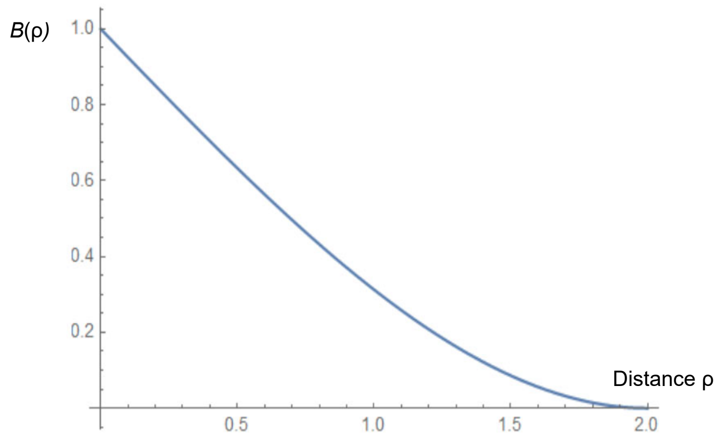
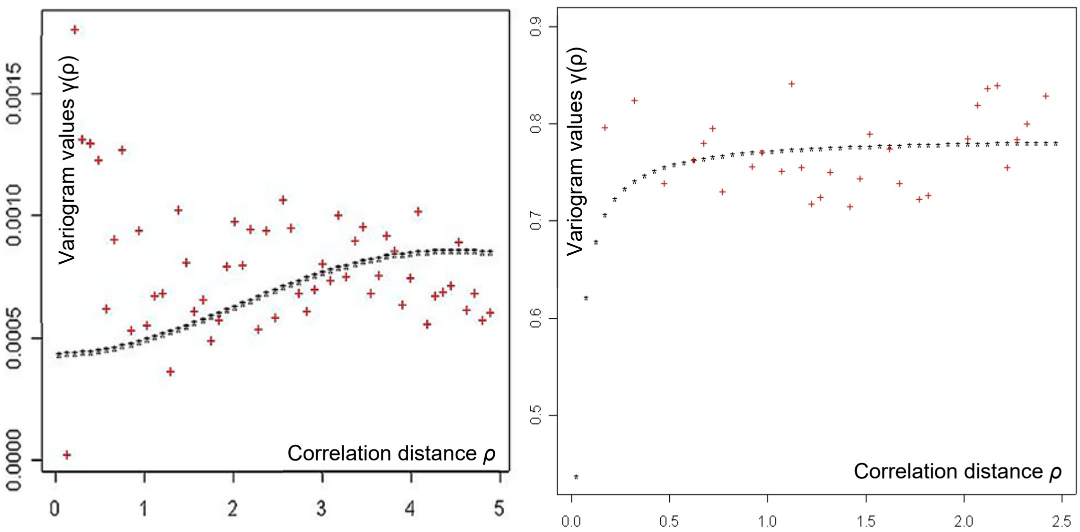


Fig. 3. The spherical type function (3) at parameter value *a* = 2



a b

Fig. 4. Empirical (rcrosses) and theoretical (curve) variograms for input data of the chalky strata, that corresponding to the: a – the Bessel (1) correlation function; b – the Cauchy (2) correlation function

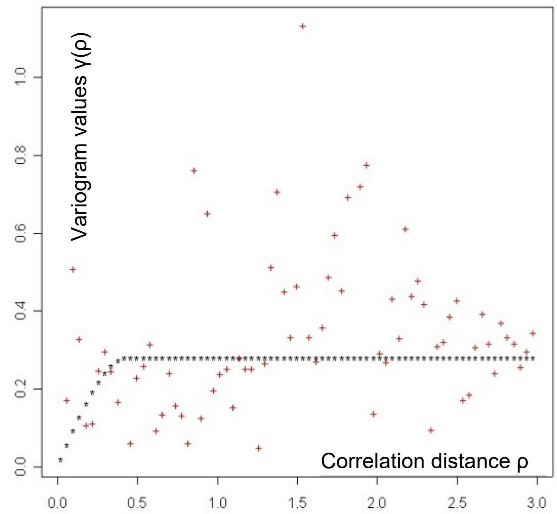


Fig. 5. Empirical (crosses) and theoretical (curve) variograms of input data arrays of chalk layer density, corresponding to sphericalcorrelation function (a ≈ 1, 25*10⁻²)

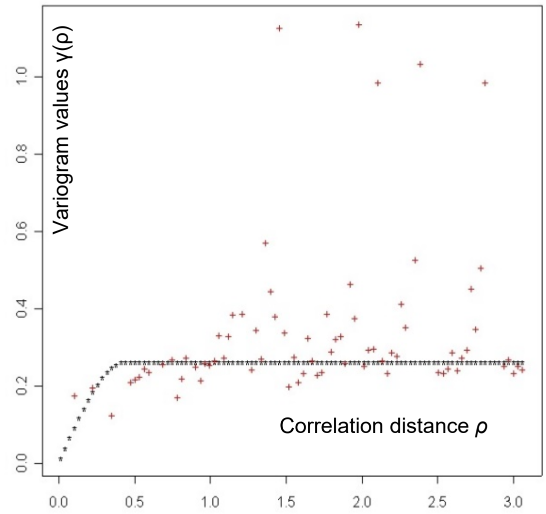


Fig. 6. Empirical (crosses) and theoretical (curve) variograms of simulated data arrays of chalk layer density, corresponding to sphericalcorrelation function (a ≈ 1, 25*10⁻²)

The spectral representation of homogeneous isotropic random fields, approximation theorem, model and statistical simulation algorithm for the spherical correlation function.

Now we will give the same theorem from spectral theory. We consider a real-valued homogeneous isotropicrandom field $\xi(r,\theta,\varphi)$ in the 3D area (r, − spherical coordinates). It is known (*Yadrenko, 1983; Vyzhva,2003; Vyzhva, 2011, p. 208*) that square-mean continuous real-valued isotropicrandom field $\xi(r,\theta,\varphi)$, that is in 3D Euclidean space R^3, admit the spectral decompositionby spherical harmonics.

The correlation function of the homogeneous isotropicrandom field $\xi(r,\theta,\varphi)$ in 3D area $B(\rho)$ depends on distance ρ between the vectors $x, y \in R^3$($x=(r,\theta_1,\varphi_1)$, $y=(r,\theta_2,\varphi_2)$) $\rho = r\sqrt{2(1-\cos\psi)} = r\sin(\psi/2)$, where $\cos\psi$ – angular distance between vectors $x, y \in R^3$:

$$\cos\psi = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2).$$

However, there is used the spectral decomposition of this random field by solution problems of statistical simulation of random field realizations in 3D space, on this figurate real-valued random variables. Let adduce that decomposition.

Theorem 1. Let a mean square continuous realvalued homogeneous isotropic random field $\xi(r,\theta,\varphi)$ is in 3D space with zero mean. Then this random field admits (*Vyzhva, 2011, p. 210*) the following spectral decomposition:

$$\xi(r,\theta,\varphi) = \\ = \sum_{m=0}^{\infty}\sum_{l=0}^{m}\tilde{c}_{m,l}P_m^l(\cos\theta)\left[\varsigma_{m,1}^l(r)\cos l\varphi + \varsigma_{m,2}^l(r)\sin l\varphi\right], \quad (4)$$

where $P_m^l(x)$ is associated Legendre functions of degree m, $\tilde{c}_{m,l}$ – constants sequences are calculated by the formula:

$$\tilde{c}_{m,l} = \tfrac{1}{2}\sqrt{\tfrac{\nu_l}{\pi}\tfrac{(m-l)!}{(m+l)!}(2m+1)},\quad \nu_l = \begin{cases}1, & l \neq 0,\\ 2, & l = 0;\end{cases} \quad (5)$$

random processes sequences $\left\{\varsigma_{m,k}^l(r)\right\}$ (k = 1, 2):

$$\varsigma_{m,k}^l(r) = \int_0^{\infty}\frac{J_{m+\frac{1}{2}}(\lambda r)}{(\lambda r)^{\frac{1}{2}}}Z_{m,k}^l(d\lambda),$$

such that satisfying the following conditions:

1) $M\varsigma_{m,k}^l(r) = 0;$
2) $M\varsigma_{m,k}^l(r)\varsigma_{m',k'}^{l'}(r) = \delta_l^{l'}\delta_m^{m'}\delta_k^{k'}b_m(r),$ (6)

where $\delta_m^{m'}$ – Kronecker symbol, $b_m(r)$ – the spectral coefficients and $\{Z_m^l(.)\}$ is a sequence of orthogonal random measures on Borel subsets from the interval $[0,+\infty)$, i. e.

$$E\,Z_m^l(S_1)\,Z_{m'}^{l''}(S_2) = \delta_l^{l'}\delta_m^{m'}\Phi(S_1\cap S_2),$$

for any Borel subsets S_1 and S_2,

where $\Phi(\lambda)$ is the bounded nondecreasing function so-called spectral function of random field $\xi(r,\theta,\varphi)$.

The spectral density of homogeneous isotropic random field $\xi(r,\theta,\varphi)$ is defined as $f(\lambda) = d\Phi(\lambda)/d\lambda$ and it is obtained bycorrelation function of this random field as integral:

$$f(\lambda) = \frac{2}{\pi}\int_0^{+\infty}\rho\lambda\sin(\lambda\rho)B(\rho)d\rho \quad (7)$$

The spectral coefficients $b_m(r)$ of random field $\xi(r,\theta,\varphi)$ are defined by the spectral density $f(\lambda)$ of this random fieldin 3D space in the way:

$$b_m(r) = \int_0^{\infty}\frac{J^2_{m+\frac{1}{2}}(\lambda r)}{\lambda r}f(\lambda)d\lambda. \quad (8)$$

Further, the statistical simulation of homogeneous isotropic random fields in the 3D space on the basis the spectral decomposition (4) coefficients (8) are considered.

Approximation **model** for the homogeneous isotropic random field $\xi(r,\theta,\varphi)$ is build by using the partial sums of series (1) and is presented by the formula:

$$\xi_N(r,\theta,\varphi) = \\ = \sum_{m=0}^{N}\sum_{l=0}^{m}c_{ml}P_m^l(\cos\theta)\left[\zeta_{m,1}^l(r)\cos l\varphi + \zeta_{m,2}^l(r)\sin l\varphi\right], \quad (9)$$

$$N \in \mathrm{N}$$

We need the mean square approximation of random field $\xi(r,\theta,\varphi)$ by model (9) in the convenient form for the constructing statistical simulation of homogeneous isotropic random field realizations in the 3D space algorithm.

Further, we used the mean square estimate from the paper (*Vyzhva et al., 2018*), that we have in following theorem.

Theorem 2. Let a mean square continuous realvaluedisotropic random field $\xi(r,\theta,\varphi)$ on the sphere $\boldsymbol{S_3}(r)$ in 3D space with zero mean (r – radius of sphere). If $\mu_3 < +\infty$, then the mean square approximation ofthis random fieldby model (9) is such that

$$M\left[\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)\right]^2 \le \frac{5\pi r^3}{2N^2}\mu_3, \quad (10)$$

where

$$\mu_3=\int_0^\infty \lambda^3\Phi(d\lambda). \quad (11)$$

If we proposed, that r (radius of sphere) is not fixed, then the random field $\xi(r,\theta,\varphi)$ is in 3D Euclidean space R^3.

Further, we described the algorithm for the statistical simulation of realizations of Gaussian homogeneous isotropic random fields $\xi(r,\theta,\varphi)$ in 3D Euclidean space R^3, which was constructed on the basis of model (9) and estimate (10)

We constructedin this paper the algorithm for the statistical simulation of Gaussian homogeneous isotropic random fieldson 3D space with spherical correlation function (3).

The spectral density is obtainedfor spherical correlation function (3) by means the formula (7) as:

$$f(\lambda)=\frac{2}{\pi\lambda}[(\frac{a+16}{2\lambda^3}-\frac{4a+1}{a\lambda})\cos(\lambda a)+ +\frac{24}{\lambda^2}(a\sin(\lambda a)-1)+\frac{3}{a}]. \quad (12)$$

The spectral coefficients, which correspond to thespherical correlation function (3) of homogeneous isotropic random field $\xi(r,\theta,\varphi)$, are calculated by the formula (7) and we have:

$$b_m(r)=\frac{2}{\pi}\int_0^\infty \frac{J^2_{m+\frac12}(\lambda r)}{\lambda^2 r}[(\frac{a+16}{2\lambda^3}-\frac{4a+1}{a\lambda})\cos(\lambda a)+\frac{24}{\lambda^2}(a\sin(\lambda a)-1)+\frac{3}{a}]\,d\lambda. \quad (13)$$

These spectral coefficients are calculated by Mathematica software for density chalky strata data.

The numerical simulation procedure of random field in 3D area with the spherical correlation function.

In this paper we generated the realizations of homogeneous isotropic random field in 3D area with the spherical correlation function (3) at the values of parameter $a \approx 1,25*10^{-2}$. The statistical simulation of density chalky strata data at the Rivne NPP object was performed by the technique of spectral decomposition and finding of spectral coefficients.

The procedure of numerical simulation the realizations of the 3D data field random component, by means of the abovementioned model (9), was conducted, which is described in (*Vyzhva et al., 2018)*.

The value of number N for the constructed model is determined by the inequality, which is the estimate of the mean square approximation of random field $\xi(r,\theta,\varphi)$ by partial sums $\xi_N(r,\theta,\varphi)$. This number N corresponds to the prescribed small number ε **(**approximation accuracy). The mentioned inequality was obtained in theorem 2. Consequently, the estimate of the mean square approximation of the random field $\xi(r,\theta,\varphi)$ with spherical typecorrelation function (3) by the partial sums $\xi_N(r,\theta,\varphi)$ has the following representation:

$$M\left[\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)\right]^2 \le \frac{5\pi r^3}{2N^2}\mu_3, \quad (14)$$

where

$$\mu_3=\frac{2}{\pi}\int_0^\infty \lambda^2\,[(\frac{a+16}{2\lambda^3}-\frac{4a+1}{a\lambda})\cos(\lambda a)+\frac{24}{\lambda^2}(a\sin(\lambda a)-1)+\frac{3}{a}]d\lambda, k=1,2.$$

We define dependence number N on r and ε in the case of spherical correlation function (3) as a following:

$$N(r,\varepsilon)\ge\sqrt{\frac{5\pi r^3}{2\varepsilon}\mu_3}. \quad (15)$$

The statistical simulation procedure of Gaussian homogeneous isotropic random field $\xi(r,\theta,\varphi)$ in 3D area with sphericalcorrelation function (3) was built by means of the model (9) and the estimate (13). This random field is determined by its statistical characteristics: the mathematical expectation and the spherical correlation function $B(\rho)$ (3) at the value of parameter $a \approx 1,25*10^{-2}$.

Algorithm.

1. Natural number N (border of summation) is chosen according to necessary accuracy $\varepsilon > 0$ of approximation the model (9) mentioned below:

$$\frac{5\pi r^3}{2N^2}\mu_3 \le \varepsilon, \quad (16)$$

where

$$\mu_3=\frac{2}{\pi}\int_0^K \lambda^2\,[(\frac{a+16}{2\lambda^3}-\frac{4a+1}{a\lambda})\cos(\lambda a)+\frac{24}{\lambda^2}(a\sin(\lambda a)-1)+\frac{3}{a}]d\lambda, K=K(a)-const.$$

2. We calculate the spectral coefficients $b_m(r)$, $m = 0, 1, 2, \ldots N$ for the spherical correlation function (3) as integral (13).

3. We simulate the sequences of independent Gaussian normalrandom variables:

$$\left\{\varsigma^l_{m,k}(r)\right\},\ k=1,2;\ m=0,1,2,\ldots N;\ l=1,\ldots,m;$$

that satisfying the following conditions (6) with spectral coefficients (13).

4. We calculate the realization of the stochastic random field $\xi(r,\theta,\varphi)$ by formula (9) in given point $(r_i,\theta_j,\varphi_p), i=1,2,\ldots,I;$ $j=1,2,\ldots,G; p=1,2,\ldots,P$ in the 3D observations area by means of substituting in it values from the previous items 1, 2 and 3, numbers N and sequences of Gaussian random variables.

5. We check whether the realization of the random field $\xi(r,\theta,\varphi)$ generated in step 4 fits the data by testing the corresponding statistical characteristics (distribution and correlation function).

The statistical simulation of realizations of the Gaussian isotropicrandom fields $\xi(r,\theta,\varphi)$ with spherical correlation functioncan be done by means of this algorithm.

Note that the procedure can be applied to random fields with another type of distribution. Then the sequences ofrandom variables $\{\zeta_{k,i}(r), i=1,2; k=0,1,2,...,N(r,\varepsilon)\}$ should be distributed by corresponding type of distribution.

The original Spectr software, which based on the results of the statistical data processing and the mentioned algorithm for the simulation values of such data realization in the 3D area, was developed in Python, where selected spherical typecorrelation function (3) was used. We calculate realizations of the random field $\xi(r,\theta,\varphi)$ in 100 points for each of 3 observations levels $(r_i,\theta_j,\varphi_p), i=1,2,...,I;$ $j=1,2,...,G; p=1,2,...,P$ in 3D are a by means of this software. The statistical estimate of the correlation function is obtained by these realizations. This estimate compares with a given spherical typecorrelation function (3) at the value of parameter $a \approx 1,25{*}10^{-2}$ and provides the statistical analysis the adequacy of realizations. The built variogram of these realizations on the study area (Fig. 6) has adequate approximation by theoretical variogram which is connected to the spherical correlation function. The results present that the chosen model of the density chalky strata data at the Rivne NPP object is adequateenough. The developed Spectr software works with sufficient accuracy.

The results, which were obtained by the simulating procedure, are displayed in Fig. 7. Fig. 7, a presents an example of constructed chalky strata density map according to observations data boreholes (averaged data over the years to 29 boreholes at 28 m) by Surfer software. Using available data the accuracy of this construction cannot provide a reliable characteristic of the chalky strata status, because the number of measurement results is not sufficient.

Fig. 7, b presents the contours of equal chalky strata density values that based on simulated data including values of the anchor boreholes by means of calculating the spectral coefficients of the spherical type. Additionally, the output data (100 simulated values in intervals between the observation points of this level) can have more reliable approximation that enables more informed decisions about the status of chalky strata and determines places for testing and additional research.

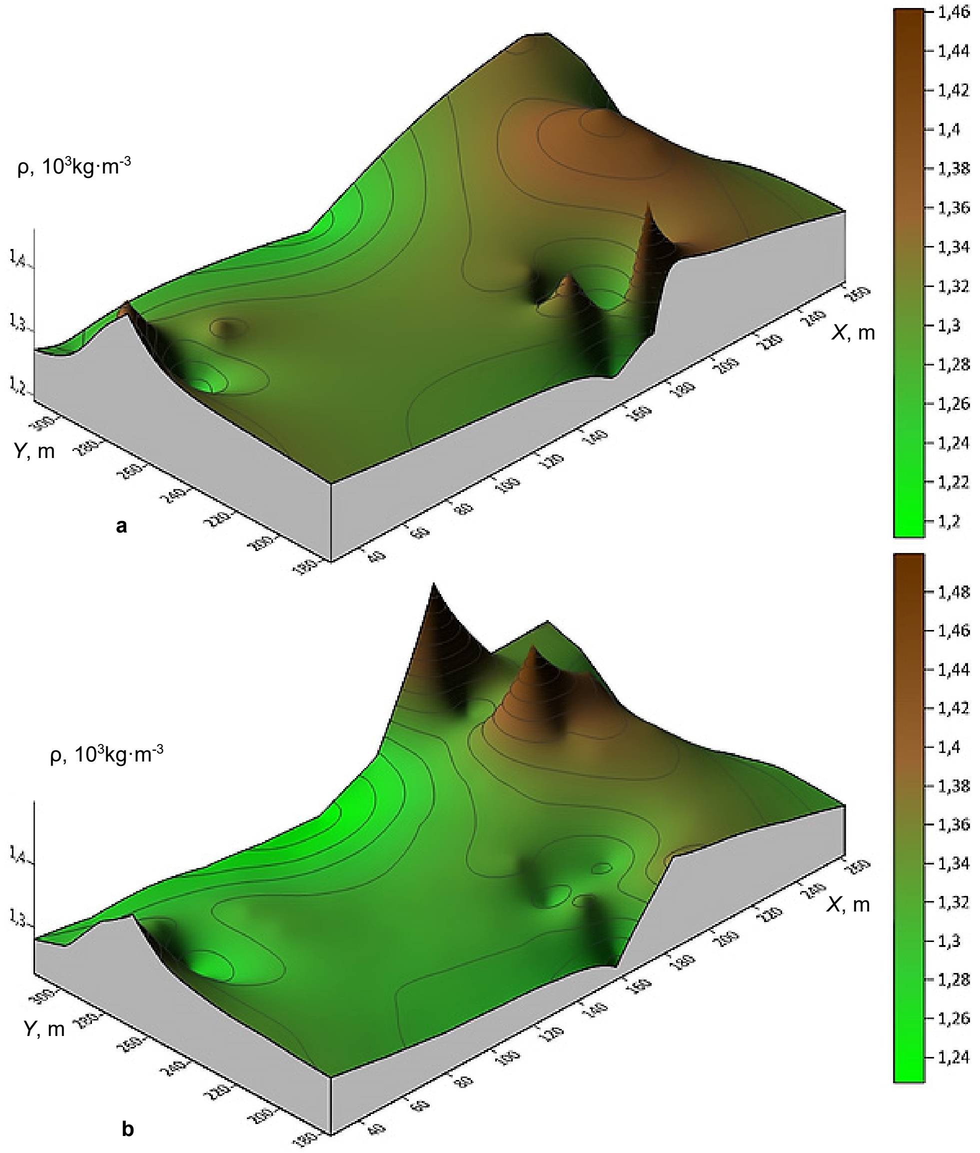


Fig. 7. The distribution of chalky strata density is on the industrial area of Rivne nuclear power plant at a depth of 28 m from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years, for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the spherical type

The following results, which were obtained by the simulating procedure for observations data boreholes (averaged data over the years to 29 boreholes at 29 m, are displayed in Fig. 8. Fig. 8, a presents an example of constructed chalky strata density map according to observations for this data by Surfer software. Fig. 8, b presents the contours of equal chalky strata density values that based on simulated data including values of the anchor boreholes. Additionally, the output data (100 simulated values in intervals between the observation points of this level) can have more reliable approximation that enables more informed decisions about the status of chalky strata.

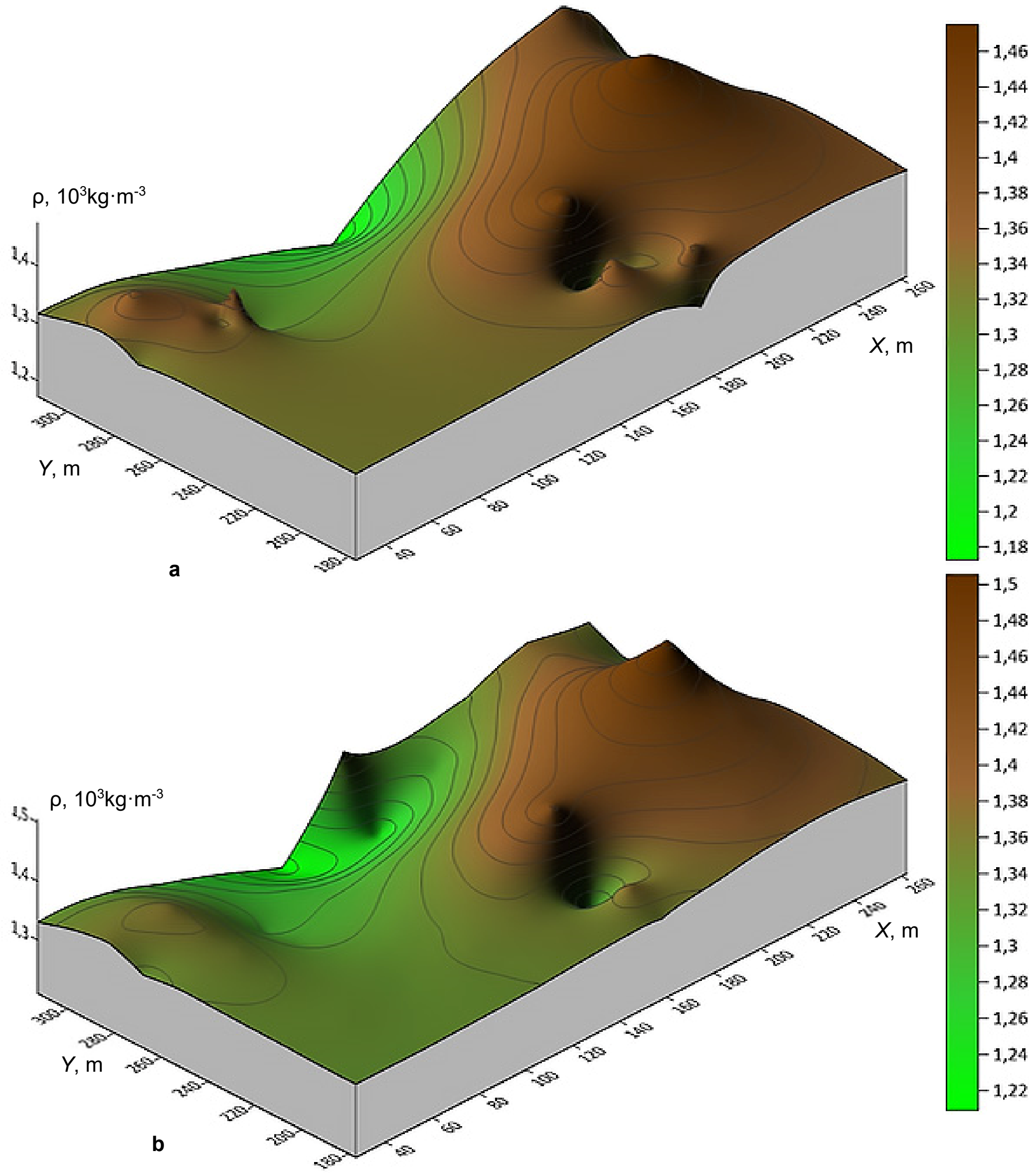


Fig. 8. The distribution of chalky strata density is on the Rivne NPP object at a depth of 29 m from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years, for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the spherical type

Finally, the results, which were obtained by the simulating procedure for observations data boreholes (averaged data over the years to 29 boreholes at 30 m, are displayed in Fig. 9. Fig. 9, a presents an example of constructed chalky strata density map according to observations for this data by Surfer software. Fig. 9, b presents the contours of equal chalky strata density values that based on simulated data including values of the anchor boreholes. Additionally, the output data (100 simulated values in intervals between the observation points of this level) can have more reliable approximation that enables more informed decisions about the status of chalky strata.

The statistical estimate of the correlation function is obtained by distribution of chalky strata density data realization in the 3D area on the Rivne NPP object. This estimate compares with a given sphericalcorrelation function (3) at the value of parameter $a \approx 1, 25*10^{-2}$ and provides the statistical adequacyanalysisof realizations.The built variogram of these realizations on the study area (Fig. 6) has adequate approximation by theoretical variogram, which is connected to the spherical correlation function. The results present that the chosen model of the data is adequate enough. The developed Spectr software works with sufficient accuracy.

Conclusions. The theory, techniques and procedure of statistical simulation of random fieldsin 3D area by using optimal in the mean square sense the spherical correlation functioncan significantly increase the effectiveness of monitoring observations on the territory of potentially dangerous objects. This makes it possible to simulate values in the area between regime observation grids and abroad,more adequately describechalky strata density 3D data on the industrial area of Rivne nuclear power plant. Because the variogram of input data has the best approximation by theoretical variogram which is connected to the spherical correlation function with a mean square deviation 0,00048, if compare the mean square approximation for Cauchy correlation function – 0,002816 and 0,0008599 – for Bessel type of correlation function.

The method of statistical simulation of random fields with thesphericalcorrelation functions allows complementing data with a given accuracy. It can also be used to detect abnormal areas.

There are many other areas of statistical simulation methods application in geosciences. Among them primary are soil science and environmental magnetism (*Menshov et al., 2015*).

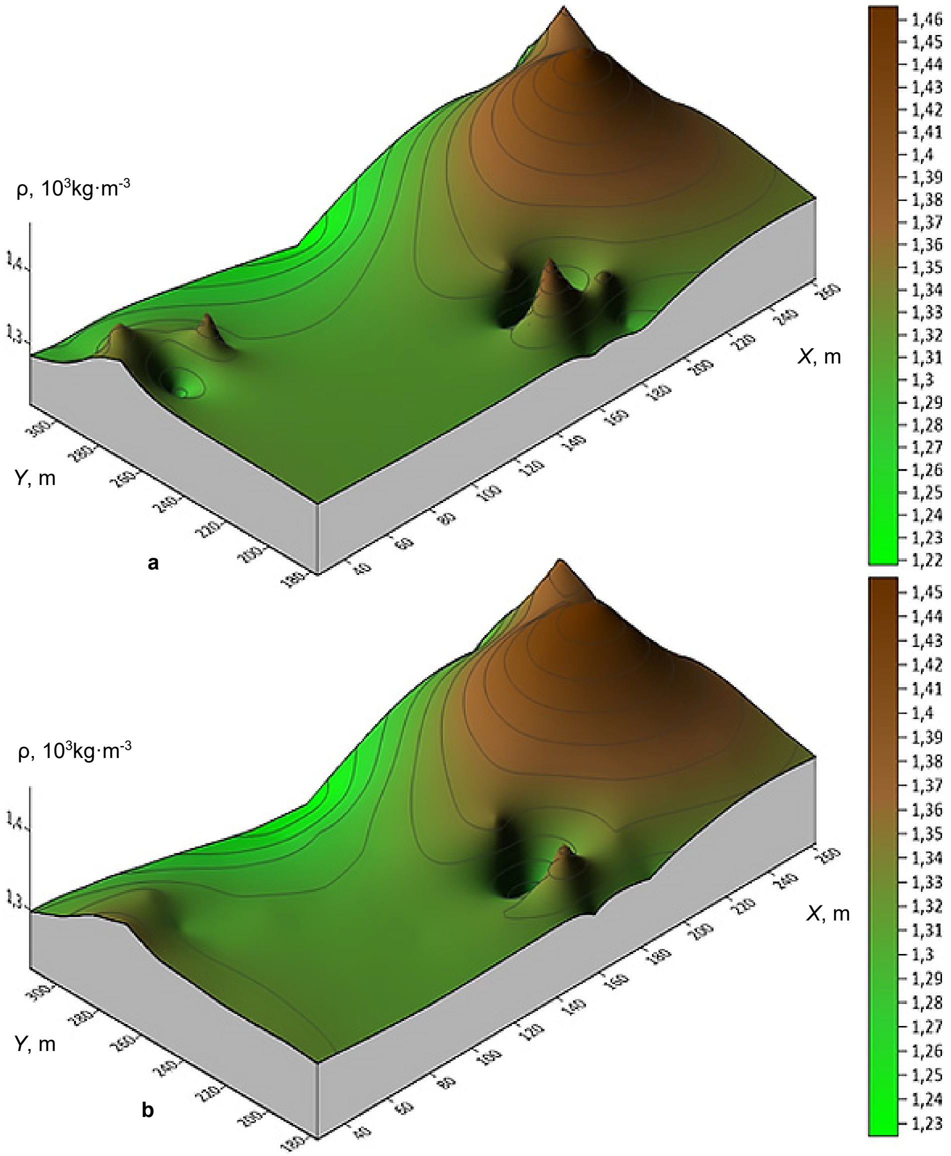


Fig. 9. The distribution of chalky strata density is on the industrial area of Rivne nuclear power plant at a depth of 30 m from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years, for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the spherical type

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З. Вижва, д-р фіз.-мат. наук, проф.,
E-mail: zoya_vyzhva@ukr.net;
В. Демидов, канд. фіз.-мат. наук, доц.,
E-mail: fondad@ukr.net;
Київський національний університет імені Тараса Шевченка,
ННІ "Інститут геології", вул. Васильківська, 90, м. Київ, 03022, Україна;
А. Вижва, канд. фіз.-мат. наук, ст. наук. співроб.,
E-mail: motomustanger@ukr.net;
ДП "Науканафтогаз"

СТАТИСТИЧНЕ МОДЕЛЮВАННЯ ДАНИХ У 3D ОБЛАСТІ ЗІ СФЕРИЧНОЮ КОРЕЛЯЦІЙНОЮ ФУНКЦІЄЮ НА ПРИКЛАДІ РІВНЕНСЬКОЇ АЕС

У зв'язку з ростом кількості природно-техногенних катастроф актуальною є розробка систем моніторингу за станом геологічного середовища з використанням сучасного математичного апарату та інформаційних технологій. У загальній системі моніторингу довкілля важливою складовою є локальний моніторинг територій розташування потенційно небезпечних об'єктів.

На території розміщення Рівненської АЕС проводився комплекс геофізичних досліджень. Серед цих моніторингових спостережень найбільший інтерес являють собою радіоізотопні дослідження густини та вологості ґрунтів по периметру збудованих споруд. При цьому виникла потреба доповнення моделюванням даних, які отримано при контролі зміни густини крейдяної товщі на території досліджуваного проммайданчика з використанням радіоізотопних методів по сітці, що включала 29 свердловин. Таку проблему було розв'язано в роботі методом статистичного моделювання, який надає можливість відображати явище (випадкове поле об'єкта дослідження в тримірній області) у будь-якій точці області спостереження. При цьому моделювалися усереднені значення густини крейдяної товщі на території проммайданчика з використанням побудованої моделі та залученням оптимальної в середньому квадратичному наближенні сферичної кореляційної функції.

Наведено розроблений алгоритм і приклад статистичного моделювання карстово-суфозійних явищ у задачі моніторингу густини крейдяної товщі на території Рівненської АЄС. За спектральним розкладом побудовано статистичну модель розподілу усередненої густини крейдяної товщі в тримірній області та розроблено алгоритм статистичного моделювання з використанням сферичної функції. На базі розробленого програмного забезпечення отримано реалізації предмета дослідження на сітці спостережень необхідної детальності та регулярності. Проведено статистичний аналіз результатів чисельного моделювання та їхня перевірка на адекватність.

Ключові слова: статистичне моделювання, сферична кореляційна функція, спектральний розклад, кондиційність карт.

З. Выжва, д-р физ.-мат. наук, проф.,
E-mail: zoya_vyzhva@ukr.net;
В. Демидов, канд. физ.-мат. наук, доц.,
E-mail: fondad@ukr.net;
Киевский национальный университет имени Тараса Шевченко,
УНИ "Институт геологии", ул. Васильковская, 90, г. Киев, 03022, Украина;
А. Вижва, канд. физ.-мат. наук, ст. науч. сотр.,
e-mail: motomustanger@ukr.net;
ГП "Науканефтегаз"

СТАТИСТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДАННЫХ В 3D ОБЛАСТИ СО СФЕРИЧЕСКОЙ КОРРЕЛЯЦИОННОЙ ФУНКЦИЕЙ НА ПРИМЕРЕ РОВЕНСКОЙ АЭС

В связи с ростом количества природно-техногенных катастроф актуальной является разработка систем мониторинга за состоянием геологической среды с использованием современного математического аппарата и информационных технологий. В общей системе мониторинга окружающей среды важная составляющая – это локальный мониторинг территорий размещения потенциально опасных объектов.

На территории расположения Ровенской АЭС проводился комплекс геофизических исследований. Среди этих мониторинговых исследований наибольший интерес представляют радиоизотопные исследования плотности и влажности грунтов по периметру построенных сооружений. При этом возникла необходимость дополнения моделированием данных, полученных при контроле изменения плотности меловой толщи на территории исследуемой промплощадки с использованием радиоизотопных методов по сетке, включающей 29 скважин. Эта проблема была решена в работе методом статистического моделирования, который даёт возможность отображать явление (случайное поле объекта исследования в трёхмерной области) в любой точке области наблюдения. При этом моделировались усреднённые значения плотности меловой толщи на территории промплощадки с использованием построенной модели с привлечением сферической корреляционной функции.

Приведен разработанный алгоритм и пример статистического моделирования карстово-суффозионных явлений в задаче мониторинга плотности меловой толщи на территории Ровенской АЭС. По спектральному разложению построена статистическая модель распределения плотности меловой толщи в трёхмерной области и разработан алгоритм статистического моделирования с использованием сферической корреляционной функции. На базе разработанного программного обеспечения получены реализации предмета исследования на сетке наблюдений в трёхмерной области необходимой детальности и регулярности. Проведен статистический анализ результатов численного моделирования и их проверка на адекватность.

Ключевые слова: статистическое моделирование, сферическая корреляционная функция, спектральное разложение, кондиционность карт.