ГЕОЛОГІЧНА ІНФОРМАТИКА

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THE STATISTICAL SIMULATION OF RANDOM FIELDS WITH THE GAUSSIAN TYPE CORRELATION FUNCTION BY THE INVESTIGATION OF THE MAGNETOMETRY DATA

(Представлено членом редакційної колегії д-ром геол. наук, ст. дослідником О.І. Меньшовим)

In the article, universal methods of statistical modeling (Monte Carlo methods) of geophysical data using the Gaussian correlation function have been developed, which make it possible to solve the problems of generating adequate realizations of random fields on a grid in three-dimensional space of required regularity and detail. Since in geophysics, most of the results of object research are presented in digital form, the accuracy of which depends on various random influences, the problem of the condition of the maps arises in the case when the data cannot be obtained with the specified detail in some observation areas. It is proposed to apply statistical simulation of random fields methods, to solve the problems of conditional maps, supplement the required detail of research results with additional data, to achieve the required accuracy of observations, and other similar problems in geophysics. An algorithm for numerical modeling of realizations of homogeneous isotropic random fields in three-dimensional space with a Gaussian correlation function is formulated on the basis of the theorem on estimation of the mean-square approximation of such random fields by the partial sum of the "spectral decomposition" series. Using the example of data from aeromagnetic surveying in the area of the Ovruch depression, the proposed algorithm for statistical modeling of random fields is implemented in solving the problems of map fitness by supplementing the data with simulated adequate implementations to the required level of detail. When analyzing data by profiles, they are divided into deterministic (trend) and random components. The trend is proposed to approximate by cubic splines and the homogeneous isotropic random component is proposed to modeling on the basis "spectral decomposition" of random fields on 3-D space in the Ovruch depression. According to the algorithm, authors received random component implementations on the study area with twice detail for each profile. When checking their adequacy, authors made the conclusions that the relevant random components histogram has Gaussian distribution. The built variogram of these implementations has the best approximation by theoretical variogram which is connected to the Gaussian type correlation function. As a result of superimposing the simulated array of the random component on the spline approximation of the real data, a more detailed implementation was obtained for the data of geomagnetic observations in the selected area. A comparative analysis of the results of modeling realizations random fields with the Gaussian correlation function with other correlation functions is carried out. Therefore, the method of statistical modeling of realizations of random fields in three-dimensional space with the Gaussian correlation function makes it possible to supplement the results of measurements of the full magnetic field intensity vector with data with a given detail as much as possible.

K e y w o r d s : ***Statistical simulation, spectral decomposition, a Gaussian correlation function, conditional maps.***

Background

The tasks of random fields statistical simulation arise solving the actual geophysics problems. The statistical simulation of random fields method is used to solve the problems of conditional maps, supplement the required detail of research results with additional data, to achieve the required accuracy of observations, and other. A special care is necessary for reduction of calculations, amount of which rapidly grow together with the dimension of the argument of the random field in this case. Many different approaches related to the solving of problems of random fields statistical simulation were described in a lot of papers, for example (Chiles, Delfiner, 2012; Vyzhva, 2011, 2021; Vyzhva et al., 2020a, 2020b, 2020c; Tolosana-Delgado, Mueller, 2021; Wackernagel, 2003).

It is proposed in the papers (Vyzhva et al., 2012, 2010; Vyzhva, Z., Vyzhva, A., 2016) to apply methods of statistical simulation of realizations of random fields on the plane (2-D space), to solve the problems of conditional maps, adding of data to achieve the necessary precision, and other similar problems in geophysics. Example for modeling is magnetometry data in those works. But the magnetometry data was investigated on 3-D space. It is divided into deterministic (trend) and random components for 3-D data analysis. The trend is proposed to approximate by cubic splines. The stationary random component of magnetometry data to modeling on the basis of spectral decomposition" of 3-D random fields with the Gaussian correlation function is proposed in this paper. The algorithm of statistical simulation of homogeneous isotropic random fields with this type correlation function on the 3-D space using approximations theorems is considered. Applying the above method makes it possible to supplement the missing magnetometry data in the study area with greater accuracy than in the paper (Vyzhva et al., 2018a) with the Bessel type correlation function.

There has been an introduced random field statistical simulation based on spectral representation in order to enhance map accuracy by the example of aeromagnetic survey data in the Ovruch depression.

Denote, that methods of statistical simulation of random field on 3-D space based on representation of it by stochastic sums was considered in papers example (Chiles, Delfiner, 2012; Vyzhva, 2003, 2011; Vyzhva et al., 2013, 2018a) and other.

The spectral representation of homogeneous isotropic random fields on 3-D space.

A real-valued homogeneous isotropic random field $\xi(r,\theta,\varphi)$ authors consider on 3-D space (r,θ,φ – spherical coordinates). On 3-D Euclidean space R^3, square-mean homogeneous isotropic random field $\xi(r,\theta,\varphi)$, which is continuous real-valued admit the spectral decomposition by spherical harmonics this result was obtained earlier (Yadrenko,1983; Vyzhva, 2003, 2011). This "spectral decomposition" is the sum:

$$\boldsymbol{\xi(r,\ \theta,\ \varphi)}\ =\ c_3\sum_{m=0}^{\infty}\sum_{l=-m}^{m}\varsigma_m^l(r)\ S_m^l(\theta,\ \varphi),\quad (1)$$

where the constant $c_3=\sqrt{2}\pi$, random processes $\varsigma_m^l(r)$ are integrals:

$$\varsigma_m^l(r)=\int_0^{\infty}\frac{J_{m+\frac{1}{2}}(\lambda r)}{(\lambda r)^{\frac{1}{2}}}\ Z_m^l(d\lambda),\quad (2)$$

where $J_{m+\frac{1}{2}}(\lambda r)$ is the Bessel function of the first kind of order $m+\frac{1}{2}$ and $\{Z_m^l(.)\}$ are the sequence of orthogonal random measures on Borel subsets from the interval $[0,+\infty)$, i. e.

$$EZ_m^l(S_1)\ Z_{m'}^{l'}(S_2)=\delta_l^{l'}\delta_m^{m'}\Phi(S_1\cap S_2),$$

for any Borel subsets S_1 and S_2, here $\delta_m^{m'}$ is Kronecker symbol, $\Phi(\lambda)$ is the bounded nondecreasing function (so-called "spectral function") and the spherical harmonics $S_m^l(x)$ are the product of functions:

$$\boldsymbol{S_m^l(\theta,\varphi)}=\tilde{c}_{m,l}\ \boldsymbol{P_m^{|l|}(\cos\theta)e^{il\varphi}},$$

where $\boldsymbol{P_m^l(x)}$ are associated Legendre functions of degree $\boldsymbol{m}$,

$$\tilde{c}_{m,l}=\frac{1}{2}\sqrt{\frac{v_l}{\pi}\ \frac{(m-l)!}{(m+l)!}\ (2m+1)},\quad (3)$$

$$v_l=\begin{cases}1, & l\neq 0,\\ 2, & l=0.\end{cases}$$

On 3-D area authors are considering the correlation function $\boldsymbol{B(\rho)}$ of the homogeneous isotropic random field $\boldsymbol{\xi(r,\theta,\varphi)}$ which depends on distance $\boldsymbol{\rho}$ between the vectors

$$\boldsymbol{x,y\in R^3}:\boldsymbol{x=(r_1,\theta_1,\varphi_1)},\qquad \boldsymbol{y=(r_2,\theta_2,\varphi_2)},$$

$$\boldsymbol{\rho=r\sqrt{2(1-\ \cos\psi)}=r\sin(\ \psi/2)},$$

where $\boldsymbol{\cos\psi}$ is angular distance between vectors $\boldsymbol{x,y\in R^3}$:

$$\boldsymbol{\cos\psi=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2)}.$$

It may by presented (*Vyzhva, 2003*) as an integral:

$$B(\rho)=\sqrt{\frac{\pi}{2}}\int_0^{\infty}\frac{J_{\frac{1}{2}}(\lambda\rho)}{\sqrt{\lambda\rho}}d\Phi(\lambda),\quad (4)$$

where $J_{\frac{1}{2}}(z)$ is the Bessel function of the first kind of order $1/2$, $\Phi(\lambda)$ is "spectral function", ρ is distance between the points $x,y\in R^3$: $x=(r_1,\theta_1,\varphi_1)$, $y=(r_2,\theta_2,\varphi_2)$.

Authors obtain the variances of random processes $\boldsymbol{\varsigma_m^l(r)}$ as:

$$b_m(r)=Var\ \varsigma_m^l(r)\ =E\left|\varsigma_m^l(r)\ \right|^2,\ l\ =\ 1,2,\ldots,h\ (m,3).$$

Then authors have the formulas for coefficients $\boldsymbol{b_m(r)}$ as an integral:

$$b_m(r)=\int_0^{\infty}\frac{J_{m+\frac{1}{2}}^2(\lambda r)}{\lambda r}\Phi(d\lambda),\ m=0,1,\ldots.\quad (5)$$

Authors will call the coefficients $\boldsymbol{b_m(r)}$ (which depends on spherical radius $\boldsymbol{r}$) as
"spectral coefficients". These coefficients are defined by the correlation function $\boldsymbol{B(\rho)}$ of the homogeneous isotropic random field in the way:

$$b_m(r)=2\pi\int_0^{\pi}B(\rho)\,P_m(\cos\psi)\sin\psi\,d\psi.\quad (6)$$

The variance of random field $\boldsymbol{\xi(r,\theta,\varphi)}$ authors obtain by this as the sum:

$$E\xi^2(r,\theta,\varphi)=Var\ \xi(r,\theta,\varphi)=\pi/2\sum_{m=0}^{\infty}(2m+1)b_m(r).\quad (7)$$

However, is used the "spectral decomposition" of homogeneous isotropic random field on 3-D space by solution statistical simulation problems of those random field's realizations and in this sum figurate real-valued random processes, which are real-valued random variables by fixed spherical radius $\boldsymbol{r}$. Let adduce that decomposition in the Theorem 1. The following statement is true.

Theorem 1. Let $\xi(r,\theta,\varphi)$ is a mean square continuous realvalued homogeneous isotropic random field in 3-D space with zero mean. Then this random field admits (*Vyzhva, 2011*) the following "spectral decomposition":

$$\xi(r,\theta,\varphi)=$$

$$=\sum_{m=0}^{\infty}\ \sum_{l=0}^{m}\tilde{c}_{m,l}\,P_m^l(\cos\theta)[\varsigma_{m,\,1}^l(r)\cos l\varphi+\varsigma_{m,\,2}^l(r)\sin l\varphi]\quad (8)$$

where $P_m^l(x)$ are associated Legendre functions of degree m, $\left\{\varsigma_{m,\,k}^l(r)\right\}$, $k=1,2$ are random velues sequences of random processes $\varsigma_m^l(r)$ as an integral (2), such that satisfying the following conditions:

$$M\varsigma_{m,\,k}^l(r)=0;\quad (9)$$

$$M\varsigma_{m,\,k}^l(r)\ \varsigma_{m',\,k'}^{l'}(r)=\delta_l^{l'}\delta_m^{m'}\delta_k^{k'}b_m(r).\quad (10)$$

In (10) $\delta_p^{p'}$ is Kronecker symbol, $\tilde{c}_{m,l}$ are constants sequences, which calculated by the formula (3), and $b_m(r)$ are the "spectral coefficients" (5).

Authors note, that the statement of Theorem 1 was proved in a number of works (Yadrenko,1983; Vyzhva, 2003).

Remark. If authors consider this theorem for the homogeneous isotropic random fields $\xi(r,\theta,\varphi)$ with Gaussian distribution, then random values sequences $\left\{\varsigma_{m,\,k}^l(r)\right\}$ in decomposition (8) are sequences of independent random values (by fixed spherical radius r) with Gaussian distribution.

The model, approximation theorems and procedure of the statistical simulation of homogeneous isotropic random fields on 3-D space.

The statistical simulation of realizations of homogeneous isotropic random fields on 3-D space on the basis of "spectral decomposition" (8) is considered.

The approximation model of homogeneous isotropic random fields $\xi(r,\theta,\varphi)$ is built using series (8) which consists of partial sums, and looks like:

$$\xi_N(r,\theta,\varphi)=\sum_{m=0}^{N}\ \sum_{l=0}^{m}\ c_{ml}\,P_m^l(\cos\theta)\left[\ \zeta_{m,\,1}^l(r)\cos l\varphi+\zeta_{m,\,2}^l(r)\sin l\varphi\right],\ N\in\mathrm{N}.\quad (11)$$

The mean square approximation of random field $\xi(\rho,\theta,\varphi)$ by model (11) is the sum:

$$M|\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)|^2\le\pi\int_0^{\infty}\left\{\sum_{m=N+1}^{\infty}\left(m+\frac{1}{2}\right)\frac{J_{m+1/2}^2(\lambda r)}{\lambda r}\right\}d\Phi(\lambda)=\pi/2\sum_{m=N+1}^{\infty}(2m+1)\int_0^{\infty}\frac{J_{m+1/2}^2(\lambda r)}{\lambda r}\ d\Phi(\lambda).$$

$$E|\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)|^2\le 2\pi\sum_{m=N+1}^{\infty}\left(m+\frac{1}{2}\right)b_m(r)\quad (12)$$

Authors need this mean square approximation in the convenient form for the constructing statistical simulation of realizations of homogeneous isotropic random fields on 3-D space algorithm. The estimates of this mean square approximation were received in the following theorems.

Authors denote the integrals as:

$$\mu_k=\int_0^{+\infty}\lambda^k\Phi(d\lambda),\ k=0,1,2,\ldots.\quad (13)$$

Theorem 2. Let a mean square continuous realvalued homogeneous isotropic random field $\xi(\rho,\theta,\varphi)$ on 3-D space with zero mean. If $\mu_3<+\infty$, then the mean square approximation of this random field by model (11) is such expression:

$$M[\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)]^2\le\frac{5\pi r^3}{2N^2}\mu_3,\quad (14)$$

where $\mu_3=\int_0^{\infty}\ \lambda^3\Phi(d\lambda)$. (15)

Authors note, that the statement of Theorem 2 was proved in the article (Vyzhva et al., 2018a).

Further authors consider another estimate of the mean square approximation of homogeneous isotropic random fields on 3-D space, where the "spectral function" $\Phi(\lambda)$ is sutisfy the following condition:

$$\mu_{2N+2} = \int_0^{+\infty} \lambda^{2N+2}\Phi(d\lambda) < +\infty.$$

Theorem 3. Let a mean square continuous realvalued homogeneous isotropic random field on 3-D space with zero mean. If $\mu_{2N+2} < +\infty$, then the mean square approximation of this random field by model $\xi_N(r,\theta,\varphi)$ (11) is such ineguality:

$$M[\xi(r,\theta,\varphi) - \xi_N(r,\theta,\varphi)]^2 \le \frac{2^{N+2}r^{2N+2}(N+1)!}{(2N+3)!}\mu_{2N+2}. \quad (16)$$

where

$$\mu_{2N+2} = \int_0^{+\infty} \lambda^{2N+2}\Phi(d\lambda). \quad (17)$$

Authors note, that the statement of Theorem 3 was proved in the article (Vyzhva et al., 2018a).

The algorithm of the statistical simulation of homogeneous isotropic random fields realizations on 3-D space may be formulated by using the approximation theorems 2 and 3, which authors were considered before. Now authors formulate the procedure of such kind, which based on the "spectral decomposition" of realvalued homogeneous isotropic random field on 3-D space (Theorem 1). For the first time this type algorithm is called "spectral coefficients algorithm" by Vyzhva Z.O. in (Vyzhva, Fedorenko, 2013a).

Below authors describe the procedure for the statistical simulation of Gaussian homogeneous isotropic random fields $\xi(r,\theta,\varphi)$ realizations on 3-D space, which was constructed on the basis of model (11) and estimates (14) and (16).

Algorithm

1. The natural number N, which is summation limit, is chosen according to necessary accuracy $\varepsilon > 0$ approximation of the model (11) by means of one of the inequalities (14) or (16). Those conditions for N, which must be fulfilled, are listed below:

$$\frac{5\pi r^3}{2N^2}\mu_3 \le \varepsilon,$$

where μ_3 is (15), or

$$\frac{2^{N+2}r^{2N+2}(N+1)!}{(2N+3)!}\mu_{2N+2} \le \varepsilon,$$

where μ_{2N+2} is (17).

2. The spectral coefficients $b_m(r)$, $m = 0,1,\ldots,N$ are calculated by formula (6) as the integral (r – fixed spherical radius).

3. Let's simulate the sequences of independent Gaussian random variables(r – fixed spherical radius):

$$\{\varsigma_{m,k}^l(r)\}, \quad k = 1,2;\ m = 0,1,2,\ldots N;\ l = 1,\ldots, m;$$

that satisfy the conditions (9) and (10).

4. Let's calculate the realization of the stochastic random field $\xi(r,\theta,\varphi)$ by formula for model (11) in given point (r_i,θ_j,φ_p), $i = 1,2,\ldots,I$; $j = 1,2,\ldots,G$; $p = 1,2,\ldots,P$ on 3-D space by means of substituting in it values from the previous items 1, 2 and 3, numbers N and sequences of Gaussin random variables.

5. Check whether the realization of the random field $\xi(r,\theta,\varphi)$ generated in step 3 fits the data by testing the corresponding statistical characteristics (distribution, correlation function $B(\rho)$).

The statistical simulation of the Gaussian homogeneous isotropic random fields realizations on 3-D space can be done by means of this algorithm. By this authors must have information about correlation function and distribution of this field. If the random field have another type of distribution (not Gaussian), then authors simulate the sequences of independent random variables in step 2 with corresponding distribution.

The statistical simulation methods of random fields by the aircraft magnetometry data on 3-D space.

The map accuracy problem occurs in geophysical research, when the data cannot be obtained with a given detail in some areas of investigation. The statistical modeling methods of random fields realizations are recommended (Vyzhva, 2003, 2011; Vyzhva et al., 2010, 2012, 2018a; Vyzhva, Fedorenko, 2013a, 2013b; Vyzhva Z., Vyzhva A., 2016) to supplement data missing in such cases.

In the presented work, the data of aeromagnetic surveys for the Ovruch depression were studied, in order to improve the accuracy of maps, on which the authors carried out statistical modeling of random fields based on the "spectral representation". The object of geophysical research was the data of aeromagnetic survey of 1:10,000 scale on the area of size 2500×2500 m^2, that was conducted during period 1996–2002.

The full magnetic field intensity vector **T** was investigated (see the map on fig. 1). The work was carried out on 25 profiles with a distance of 100 meters between them (X from 0 to 2500 m and Y from 0 to 2500 m) and authors have for statistical analyses 625 points of investigation.

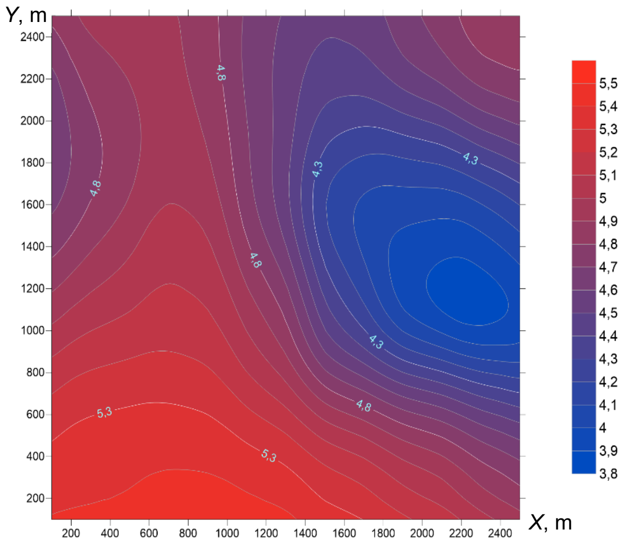

Fig. 1. The map of aeromagnetic survey data ΔT_{an} in the Ovruch depression (built in Surfer)

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Further authors translate the Cartesian coordinates (x, y, z) of the three-dimensional space, which are tied to the points of measurement, into spherical coordinates (r, θ, φ) for using our random fields statistical modeling method.

Authors did the data analysis, while constructing data graphs for input data of each profile and authors think that it is expedient to distinguish deterministic and random components. The deterministic function (trend $f_i(r, \theta, \varphi), i = 1, 2, ..., 25$ – profile numbers) can be selected in different ways. One determination method was considered in (Vyzhva et al., 2012). But there is a more accurate way to select deterministic component – approximation by cubic spline data (Vyzhva et al., 2010, 2018a; Vyzhva, Z., Vyzhva A., 2016). The difference between spline approximation of data and input data is a random process that is frequently stationary for most profiles of investigation.

Authors use the notation of the input data on the profile as a random field $\eta_i(r, \theta, \varphi)$, where i is profile numbers. The stationary random component $\xi_i(r, \theta, \varphi)$ (random fields) for input data and trend $f_i(r, \theta, \varphi)$ as determined cubic spline function were selected for each profile ($i = 1, 2, ..., 25$). Thus, the input data on profiles is a random field in the form of a sum:

$$\eta_i(r; \theta, \varphi) = f_i(r, \theta, \varphi) + \xi_i(r, \theta, \varphi), \ i = 7, \dots, 20. \quad (18)$$

Now authors introduce the notation of spline approximation for input data in the profile number i as $S_i^{(1)}(r, \theta, \varphi)$, built by means of the MathCad software for PRi (profile № i) data. Parameters defined by the data were determined for such spline in the profile.

Based on observations (values) of random component $\xi_i(r, \theta, \varphi)$, $i = 7, \dots, 20$ in all 13 profiles authors created array that frequently represents isotropic random field $\xi(r, \theta, \varphi)$ on 3-D space with zero mathematical expectation and approximately Gaussian distribution (fig. 2).

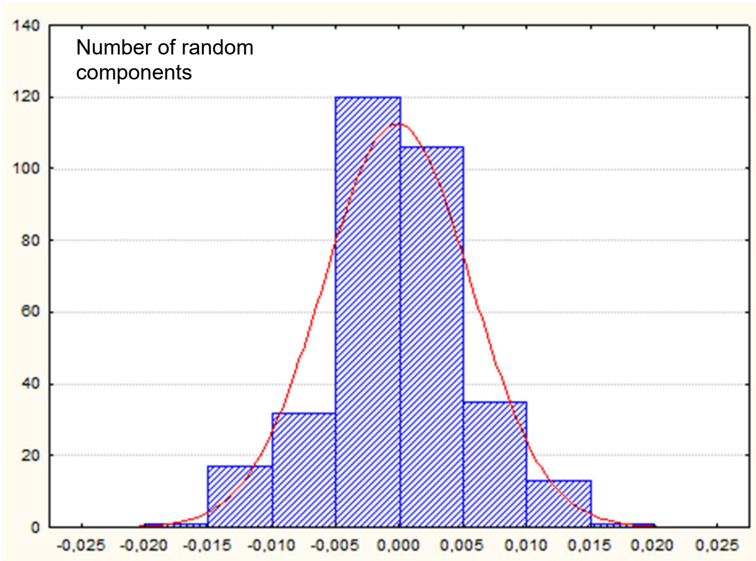

Fig. 2. Histogram for observed values of random component for input aeromagnetic survey data in all 13 profiles (for PR7-PR20) in the Ovruch depression. The red line indicates the density of the Gaussian distribution

By availability of such properties of input data, authors can apply the method of statistical simulation of random fields on 3-D space based on their "spectral decomposition" (8) for aeromagnetic survey data, which allows finding the perfect image of entire observations field for their certain implementation values. So, authors generate additional random component data in the points from investigation areas on 3-D space where geomagnetic measurements were not carried out, for example, with double precision intervals of 50 compare to 100 meters or between profiles. Authors can impose this modeling data on the spline curve trend $S_i^{(1)}(r, \theta, \varphi), i = 7, \dots, 20$ for each profile and obtain more detailed aeromagnetic survey data in the field of observations. This method differs from the traditional interpolation method, which uses the average of neighboring measured points for the calculation point. Our method takes into account the statistical distribution of airborne magnetic survey data and the correlation between data points. Using the above method makes it possible to supplement the missing data in the study area with greater accuracy than in (Vyzhva, Fedorenko, 2013b). (the mean square deviation is 0, 225) and then in (Vyzhva et al., 2018a) with Bessel type correlation function (the mean square deviation is 0, 195), taking into account their statistical nature.

The built variogram of these implementations $\xi_i(r, \theta, \varphi), i = 7, \dots, 20$ has the best approximation (the mean square deviation is 0,011) by theoretical variogram which is connected to the Gaussian type correlation function (Vyzhva, 2003) for parameter $c \approx 4{,}2 * 10^{-3}$. In this case, the Gaussian type correlation function has form:

$$B(\rho) = exp\ \{-c\ \rho^2\}, c > 0. \quad (19)$$

This confirms the adequacy of simulated implementations to the real research data.

Based on this article, the authors have built an improved algorithm for the statistical simulation of Gaussian isotropic random fields on 3-D space with Gaussian type correlation function.

The "spectral density" is obtained by Gaussian type correlation function (19) as following formula:

$$f(\lambda) = \frac{\lambda^2}{2\sqrt{\pi c^3}} exp\ \left\{-\frac{\lambda^2}{4c}\right\}. \quad (20)$$

According to formula:

$$b_m(r) = \left(\frac{\pi}{c}\right)^{\frac{3}{2}} \frac{1}{2r} exp\ \{-2rc^2\} I_{m+\frac{1}{2}}(2cr^2) \quad (21)$$

are calculated the "spectral coefficients", which correspond to the Gaussian type correlation function (19) and spectral density (20) formula of random field $\xi(r, \theta, \varphi)$, where $I_m(z)$ is the modified Bessel function of the first kind of order m.

These "spectral coefficients" authors used in proposed above algorithm. The statistical simulation of realizations of the homogeneous isotropic random fields $\xi_i(r,\theta,\varphi), i=7,...,20$ on 3-D space with Gaussian type correlation function can be done by means of constructed algorithm.

Earlier, based on the estimate from (Vyzhva et al., 2018b), and model (11), an algorithm for statistical modeling of realizations of Gaussian homogeneous isotropic random fields was described. In the case of the Bessel-type correlation function, this algorithm was constructed in following (Vyzhva et al., 2018a, 2018b). In the articles (Vyzhva et al., 2019, 2020) the considered algorithm was constructed for the spherical correlation function and in the paper (Vyzhva et al., 2020b) this algorithm was constructed for "cubic" correlation function.

Below authors describe the procedure for the statistical simulation of Gaussian homogeneous isotropic random fields $\xi(r,\theta,\varphi)$ realizations on 3-D space for random fields with Gaussian type correlation function.

The algorithm for random fields with Gaussian type correlation function.

1. The natural number N, which is summation limit, is chosen according to the required accuracy $\varepsilon > 0$ of the approximation of the model (11). In this case, for the Gaussian type correlation function "spectral coefficients" (21) of the model (11), the condition is fulfilled:

$$\frac{5\pi r^3}{2N^2}\mu_3 \le \varepsilon \qquad (22)$$

where

$$\mu_3 = 1/2\sqrt{\pi c^3}\int_0^\infty \lambda^5 exp\,(-\frac{\lambda^2}{4c})d\lambda\ .$$

2. For the Gaussian type correlation function (19), the "spectral coefficients" $b_m(r),\ m=0,1,2,...N$ are calculated as the integral (21).

3. Let's model the sequences of independent Gaussian normal random variables which have the form $\{\varsigma^l_{m,k}(r)\}$, $k=1,\ 2;\ m=0,1,2,\dots N;\ l=1,\dots,\ m;$ which satisfying the conditions (22) with "spectral coefficients" (21).

4. By substituting the calculating number N, "spectral coefficients" values $b_m(r),\ m=0,1,2,...N$, for the Gaussian type correlation function $\{\varsigma^l_{m,k}(r)\},\ k=1,2;\ m=0,1,2,...N;$ $l=\ 1,\dots,\ m;$ and obtained in the previous items 3 sequences of Gaussian random variables into formula (11) for the model, authors calculate the realization value of the random field $\xi(r,\theta,\varphi)$ at the given point on 3-D space: (r_i,θ_j,φ_p), $i=1,2,\dots,I;\ j=1,2,\dots,G;\ p=1,2,\dots,P.$

5. Authors check whether the realization of the random field $\xi(r,\theta,\varphi)$ generated in step 4 fits the data by testing the corresponding statistical characteristics (distribution and Gaussian type correlation function).

The implementation of the above algorithm makes it possible to more accurately fill in the missing aeromagnetic survey data in the study area in the Ovruch depression. The authors generate additional data realizations of the random component in the points from investigation areas where geomagnetic measurements were not carried out (with a double precision interval of 50 meters compared to 100 meters between profiles). Next, a statistical analysis of implementations modeled according to this algorithm is carried out. For this purpose, semivariograms of arrays of input and simulated data were constructed.

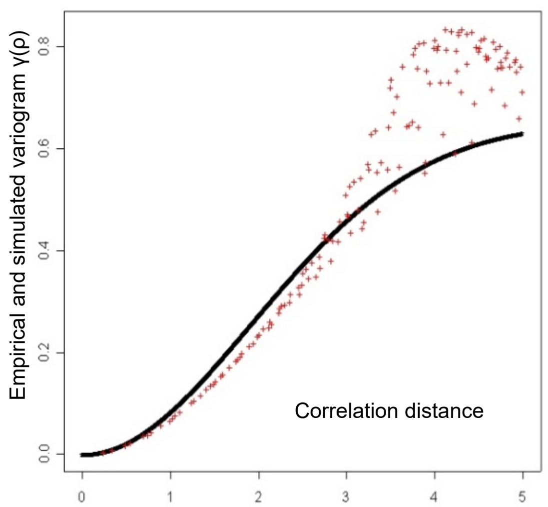

Fig. 3. Empirical (black line) and simulated (red crosses) variograms of input data arrays ΔTan for PR7-PR20, which corresponding to Gaussian type correlation function $B(\rho)=exp\ \{-c\rho^2\}\ (c\approx 4,2*10^{-3})$

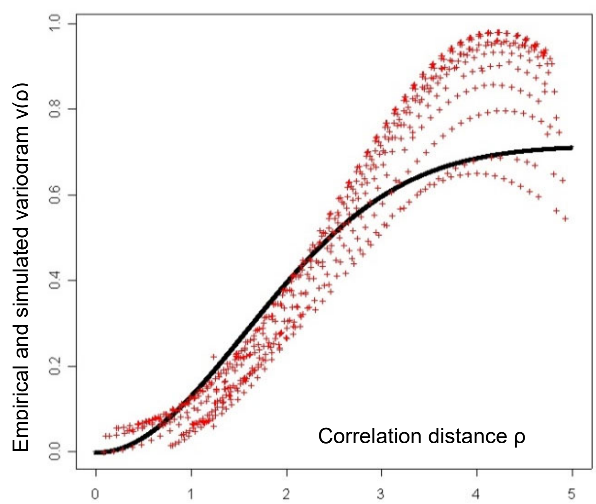

Fig. 4. Empirical (black line) and simulated (red crosses) variograms of simulated data arrays ΔTan for PR7-PR20, which corresponding to Gaussian type correlation corresponding to Gaussian type correlation function $B(\rho)=exp\ \{-c\rho^2\}\ (c\approx 4,2*10^{-3})$

The variograms of input and simulated data arrays ΔT_{an} for PR7-PR20, corresponding to Gaussian type correlation function (25) at the value of the parameter $c\approx 4{,}2*10^{-3}$ are shown in fig. 3 and fig. 4 respectively. This confirms the adequacy of simulated by this algorithm implementations to the real aeromagnetic survey research data. So, authors generated the adequate random component implementations on the study area in the Ovruch depression with twice detail for each profile, according to this algorithm.

The final stage of our method was the superimposing of realizations array $\xi_i(r,\theta,\varphi),\ i=7,\dots,20$, that we got by statistical simulation, on the spline approximation $S^{(1)}_i(r,\theta,\varphi),\ i=7,\dots,20$ of real aeromagnetic survey data. Finally, authors received more detailed implementation for the geomagnetic observation data in the Ovruch depression selected area as a result of our modeling work. Authors built the map (a) of aeromagnetic survey data ΔT_{an} (general map) and the map (b) of aeromagnetic survey data ΔT_{an} with generated by the Gaussian type correlation function additional data in the points with double precision intervals in the Ovruch depression (fig. 5).

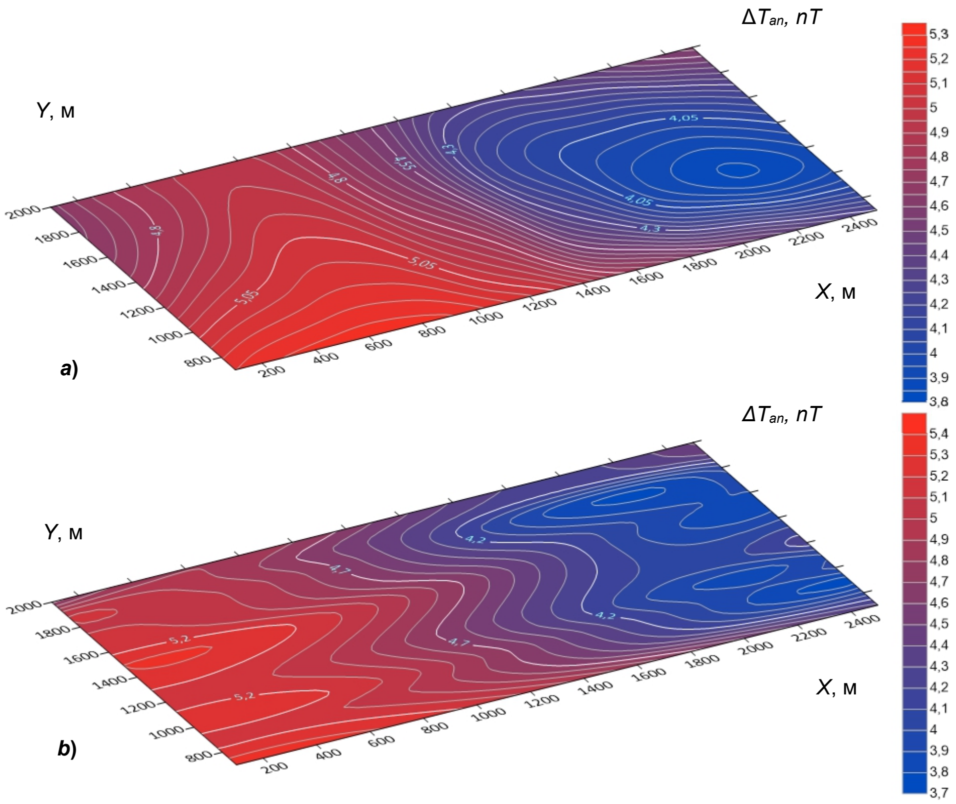

Fig. 5. *a*) The map of aeromagnetic survey data ΔT_{an} (general map) M 1:10 000, (PR 7-20);
***b*) the map of aeromagnetic survey data ΔT_{an} plus generated with the Gaussian type correlation function additional data in the points with double precision intervals in the Ovruch depression M 1:10 000 (built in Surfer)**

Conclusions

The method of statistical simulation of a random field on spatial three-dimensional realizations makes it possible to supplement the measurement results of the full vector of the magnetic field over a great square area in the Ovruch depression with a given detail.

The built variogram of random component for aeromagnetic survey data has the best approximation by theoretical variogram which is connected to the Gaussian type correlation function (the mean square deviation is 0,011), then in (Vyzhva et al., 2010; Vyzhva, Z., Vyzhva A., 2016) which is connected to the Bessel type correlation function (the mean square deviation is 0,225).

The algorithm for statistical simulation of realizations of Gaussian homogeneous isotropic random fields in three-dimensional space with a Gaussian-type correlation function proposed for use in this paper is an important addition to the Monte Carlo method used in geophysics. It can also be used to detect abnormal areas**.**

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СТАТИСТИЧНЕ МОДЕЛЮВАННЯ ВИПАДКОВИХ ПОЛІВ ІЗ ГАУССІВСЬКОЮ КОРЕЛЯЦІЙНОЮ ФУНКЦІЄЮ ДЛЯ ДОСЛІДЖЕННЯ ДАНИХ МАГНІТОМЕТРІЇ

Розроблено універсальні методи статистичного моделювання (методи Монте-Карло) геофізичних даних із застосуванням Гауссівської кореляційної функції, які дають змогу розв'язати проблеми генерування адекватних реалізацій випадкових полів на сітці в тривимірному просторі будь-якої регулярності та детальності. Оскільки в геофізиці більшість результатів досліджень об'єктів подається у цифровій формі, точність якої залежить від різних випадкових впливів, то при цьому виникає проблема кондиційності карт у випадку, коли дані неможливо отримати із заданою детальністю в деяких ділянках спостережень. Для розв'язання проблем кондиційності карт, доповнення додатковими даними потрібної детальності результатів досліджень, для досягнення необхідної точності спостережень та інших проблем подібного роду в геофізичних задачах пропонується застосовувати методи статистичного моделювання випадкових полів.

Сформульовано алгоритм чисельного моделювання реалізацій однорідних ізотропних випадкових полів у тривимірному просторі з Гауссівською кореляційною функцією на основі теореми про оцінку середньоквадратичної апроксимації таких випадкових полів частковою сумою ряду "спектрального розкладу". На прикладі даних аеромагнітної зйомки в районі Овруцької западини впроваджено запропонований алгоритм статистичного моделювання випадкових полів у розв'язанні проблем кондиційності карт шляхом доповнення даних змодельованими адекватними реалізаціями до необхідної детальності. Під час аналізу даних по профілях їх розділено на детерміновану (тренд) та випадкову складові. Тренд даних пропонується наближати кубічними сплайнами, однорідну ізотропну випадкову складову – моделювати на основі "спектрального розкладу" випадкових полів у тривимірному просторі. Модельний приклад – дані аеромагнітної зйомки на території Овруцької западини. За наведеним алгоритмом було отримано реалізації випадкової складової в області дослідження із подвоєною детальністю по кожному профілю. Перевіряючи їх на адекватність, зроблено висновки, що відповідна гістограма випадкової складової має гауссівський розподіл. Побудована варіограма цих реалізацій має найкраще наближення теоретичною варіограмою, яка пов'язана з кореляційною функцією Гауссівського типу. У результаті накладення змодельованого масиву випадкової складової на сплайнову апроксимацію реальних даних отримано більш детальну реалізацію для даних геомагнітних спостережень у виділеній області. Проведено порівняльний аналіз результатів моделювання реалізацій випадкових полів із Гауссівською кореляційною функцією з іншими кореляційними функціями. Отже, метод статистичного моделювання реалізацій випадкових полів у тривимірному просторі з Гауссівською кореляційною функцією дає можливість максимально адекватно доповнити даними із заданою детальністю результати вимірювань повного вектора напруженості магнітного поля.

Ключові слова: *статистичне моделювання, спектральний розклад, Гауссівська кореляційна функція, кондиційність карт.*

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